Math Test 1

Subjective Test

(i) All questions are compulsory.
*(ii) Questions **1** to **10** are very short answer type questions. These questions carry one mark each.*
*(iii) Questions **11** to **15** are short answer type questions. These questions carry two marks each.*
*(iv) Questions **16** to **25** are also short answer type questions. These questions carry three marks each.*
*(v) Questions **26** to **30** are long answer type questions and carry six marks each.*

Question 1 (1.0 marks)

Find the value of *m* in the expression $\dfrac{3^{7+m}}{(9)^{-6}\times(81)^{2}}=(243)^{3}$

Solution:

$$\frac{3^{7+m}}{(9)^{-6}\times(81)^{2}}=(243)^{3}$$

$$\frac{3^{7+m}}{(3^{2})^{-6}\times(3^{4})^{2}}=(3^{5})^{3}$$

$$\frac{3^{7+m}}{3^{-12}\times 3^{8}}=3^{15} \qquad (a^{m})^{n}=a^{m\times n}$$

$$\frac{3^{7+m}}{3^{-12+8}}=3^{15} \qquad (a^{m}\times a^{n}=a^{m+n})$$

$$\frac{3^{7+m}}{3^{-4}}=3^{15}$$

$$3^{7+m+4}=3^{15} \qquad \left(\frac{a^{m}}{a^{n}}=a^{m-n}\right)$$

$$3^{11+m}=3^{15}$$

$$11+m=15 \qquad (a^{m}=a^{n}\Rightarrow m=n)$$

$$\therefore m=15-11=4$$

Question 2 (1.0 marks)

Find 5 rational numbers between $\dfrac{5}{7}$ and $\dfrac{4}{5}$.

Solution:

$\dfrac{5}{7}$ and $\dfrac{4}{5}$ can be written as

$$\frac{5}{7}=\frac{5\times10}{7\times10}=\frac{50}{70}$$

$$\frac{4}{5}=\frac{4\times14}{5\times14}=\frac{56}{70}$$

Thus, 5 rational numbers between $\dfrac{5}{7}$ and $\dfrac{4}{5}$ are $\dfrac{51}{70},\dfrac{52}{70},\dfrac{53}{70},\dfrac{54}{70},\dfrac{55}{70}$

Question 3 (1.0 marks)

Find a Pythagorean triplet, one of whose member is 530.

Solution:
Let $m^2 - 1 = 530$

$\Rightarrow m^2 = 531$

Therefore, the value of m will not be an integer.

Let us take $m^2 + 1 = 530$

$\Rightarrow m^2 = 529$

$\therefore m = 23$

$\Rightarrow 2m = 2\ (23) = 46$

$m^2 - 1 = (23)^2 - 1 = 529 - 1 = 528$

Thus, the Pythagorean triplet is 46, 528, and 530.

Question 4 (1.0 marks)
Find the smallest perfect square number which is a multiple of 21, 28, and 45.

Solution:
The prime factorisation of 21, 28, and 45 is as follows.

$21 = 7 \times 3$

$28 = 7 \times 2 \times 2$

$45 = 5 \times 3 \times 3$

$\therefore$ L.C.M. (21, 28, 45) $= 2 \times 2 \times 3 \times 3 \times 5 \times 7$

It can be seen that the L.C.M. of these numbers is not a perfect square. Therefore, to make it a perfect square, it is multiplied by 5 and 7.

Thus, the smallest perfect square number which is a multiple of 21, 28, and 45 is

$(2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 7 \times 7) = 44100$

Question 5 (1.0 marks)
Is it possible to draw a regular polygon whose exterior angle is 22.5°? If yes, then how many sides will the polygon have? Also, find the sum of all interior angles of this polygon.

Solution:
We know that the sum of the exterior angles of a regular polygon is 360°.

For an n-sided polygon, there will be n exterior angles.

$\therefore n \times 22.5° = 360°$

$\Rightarrow n = \dfrac{360°}{22.5°} = 16$

Thus, the polygon will have 16 sides.

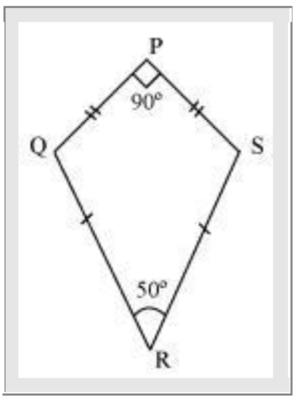
Sum of all interior angles of an n-sided polygon = $(n - 2) \times 180°$

$= (16 - 2) \times 180°$

$= 14 \times 180°$

$= 2520°$

Question 6 (1.0 marks)
Use the following information to answer the next question.



Find the measure of ∠PQR and ∠PSR for the given figure.

Solution:
It can be seen that the given figure is a kite with PQ = PS and QR = RS.

∴ ∠PQR = ∠PSR

The sum of all interior angles of a quadrilateral is 360°.

⇒ ∠PQR + ∠QRS + ∠RSP + ∠SPQ = 360°

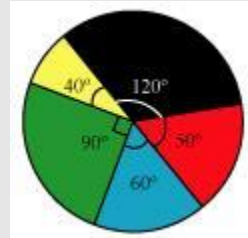
2∠PQR + 50° + 90° = 360°

∠PQR = 110°

∴ ∠PQR = ∠PSR = 110°

Thus, the measure of ∠PQR and ∠PSR is 110°.

Question 7 (1.0 marks)
Use the following information to answer the next question.

A spinning wheel is divided into 5 regions as shown in the figure. If the pointer wheel falls in either blue or yellow region, then the player will win.	

Find the probability that a player will win.

Solution:
The blue and yellow region covers an area of (60° + 40°) = 100° out of 360°.

Probability of winning the game = $\frac{100°}{360°} = \frac{5}{18}$

Question 8 (1.0 marks)
The sum of 3 consecutive common multiples of 5 and 7 is 525. Find these multiples.

Solution:
We know that the L.C.M. of 5 and 7 is 35.

Every multiple of 35 will be a common multiple of 5 and 7 both.

Let these multiples be *x*, *x* + 35, and *x* + 70.

$$x + x + 35 + x + 70 = 525$$
$$3x + 105 = 525$$
$$3x = 525 - 105 = 420$$
$$x = \frac{420}{3} = 140$$
$$x + 35 = 140 + 35 = 175$$
$$x + 70 = 140 + 70 = 210$$

Thus, the common multiples are 140, 175, and 210 respectively.

Question 9 (1.0 marks)
Evaluate by using an identity: 984 × 1016

Solution:
984 and 1016 can be written as (1000 − 16) and (1000 + 16) respectively.

∴ 984 × 1016 = (1000 − 16) (1000 + 16)

= $(1000)^2 - (16)^2$

= 1000000 − 256

= 999744

Question 10 (1.0 marks)

Factorize $x^2+\frac{1}{x^2}-23.$

Solution:

$x^2+\frac{1}{x^2}-23=x^2+\frac{1}{x^2}+2-23-2$

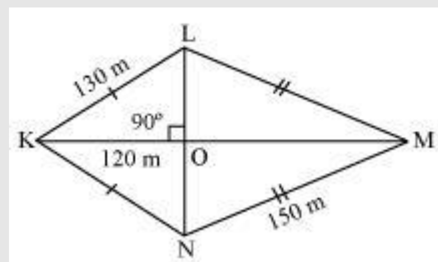
$=\left(x^2+\frac{1}{x^2}+2\right)-25$

$=\left[x^2+\frac{1}{x^2}+2\times x\times\frac{1}{x}\right]-(5)^2=\left(x+\frac{1}{x}\right)^2-(5)^2$

$=\left(x+\frac{1}{x}-5\right)\left(x+\frac{1}{x}+5\right)$

Question 11 (2.0 marks)

Use the following information to answer the next question.

A park is in the shape of a kite as shown in the figure. KM and LN are two paths in the park which meet each other at O. $\left(\text{Assume } \sqrt{2}=1.414\right)$	

Find the length of roads KM and LN.

Solution:

The given park is in the shape of a kite.

∴ ∠LOK = 90°

LO = ON

Using Pythagoras' theorem in ΔLOK, we obtain

$LK^2 = LO^2 + OK^2$

$(130\text{ m})^2 = (LO)^2 + (120\text{ m})^2$

$16900\text{ m}^2 - 14400\text{ m}^2 = (LO)^2$

$(LO)^2 = 2500\text{ m}^2 = (50\text{ m})^2$

∴ LO = 50 m

$\Rightarrow$ LN = 2 × LO = 2 × 50 = 100 m

Similarly, in ΔNOM,

$(NO)^2 + (OM)^2 = (NM)^2$

$(50\text{ m})^2 + (OM)^2 = (150\text{ m})^2$

$2500\text{ m}^2 + (OM)^2 = 22500\text{ m}^2$

$OM^2 = 20000\text{ m}^2$

$\Rightarrow$ OM = $100\sqrt{2}$ m

$\therefore$ KM = KO + OM $= 120\text{ m} + 100\sqrt{2}\text{ m}$ = 120 m + 100 m × 1.414 = 261.4 m

Thus, the length of KM and LN are 261.4 m and 100 m respectively.

Question 12 (2.0 marks)
Three numbers are in the ratio 1: 6: 8. The sum of their cubes is 5832. Find the square of their sum.

Solution:
Let the numbers be x, $6x$, and $8x$.

According to the given information,

$(x)^3 + (6x)^3 + (8x)^3 = 5832$

$x^3 + 216x^3 + 512x^3 = 5832$

$729x^3 = 5832$

$$\Rightarrow x^3 = \frac{5832}{729} = 8$$
$$\Rightarrow x^3 = 2\times2\times2$$
$$\Rightarrow x = \sqrt[3]{2\times2\times2} = 2$$

Therefore, the numbers are

$x = 2$,

$6x = 6 \times 2 = 12$, and

$8x = 8 \times 2 = 16$

Square of the sum of the numbers = $(2 + 12 + 16)^2 = 30^2 = 900$

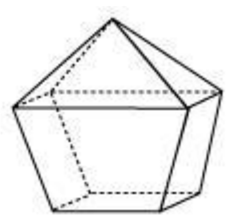
Question 13 (2.0 marks)

(a)The following table represents the number of faces (F), vertices (V), and edges (E) of some polyhedrons. Complete the following table.

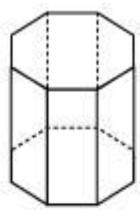
F	?	8
V	16	?
E	24	18

(b)Verify Euler's formula for the following solids.

(i)



(ii)



Solution:
(a) Euler's formula for polyhedrons states that F + V − E = 2, where F is the number of faces, V is the number of vertices, and E is the number of edges of the polyhedron.

Therefore, for V = 16 and E = 24,

$F + V - E = 2$

$\Rightarrow F + 16 - 24 = 2$

$\Rightarrow F = 10$

For F = 8 and E = 18,

$F + V - E = 2$

$\Rightarrow 8 + V - 18 = 2$

$\Rightarrow V = 12$

(b) (i) In the given figure, V = 9, F = 9, and E = 16

∴ F + V − E = 9 + 9 − 16 = 2

Hence, Euler's formula is verified.

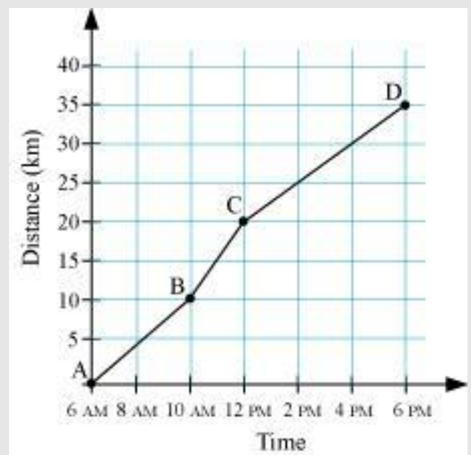
(ii) V = 16, E = 24, and F = 10

∴ F + V − E = 10 + 16 − 24 = 2

Hence, Euler's formula is verified.

Question 14 (2.0 marks)
Use the following information to answer the next question.

The given graph represents the distance travelled by a cyclist and the time taken by him.



(a)Find the maximum speed of the cyclist during his journey from A to D.

(b)If the cyclist travelled with the uniform speed obtained in the above question, then find the time taken by him to reach a place which is at a distance of 80 km from point A.

Solution:
(a)We know that, speed is given by. $\frac{\text{Distance}}{\text{Time}}$

∴ Speed of cyclist between point A and B $= \frac{10}{4} = 2.5 \text{ km/hr}$

Speed of cyclist between point B and C $= \frac{10}{2} = 5 \text{ km/hr}$

Speed of cyclist between point C and D $= \frac{15}{6} = 2.5 \text{ km/hr}$

It can be seen that the speed was maximum between point B and C.

(b)Distance travelled = 80 km

Time taken $= \frac{80}{5} = 16 \text{ h}$

Thus, the time taken to travel 80 km by the cyclist is 16 hours.

Question 15 (2.0 marks)
The denominator of a rational number is 4 more than 3 times of its numerator. When 2 is added to its denominator, the number becomes.$\frac{1}{5}$

Find the rational number.

Solution:
Let the numerator of this number be *x*.

∴ Denominator = 4 + 3 (Numerator) = 4 + 3(x) = $3x$ + 4

$$\frac{(x)}{(3x+4)+2}=\frac{1}{5}$$
$$5x=3x+4+2$$
$$5x=3x+6$$
$$2x=6$$
$$x=3$$

∴Denominator $=3(3)+4=13$

Thus, the required rational number is.$\frac{3}{13}$

Question 16 (3.0 marks)
Construct a quadrilateral MONA, where MO = 4 cm, ON = 6 cm, ∠M = 120°, ∠N = 60°, and ∠A = 150°.

Solution:
We know that interior angle sum of a quadrilateral is 360°.

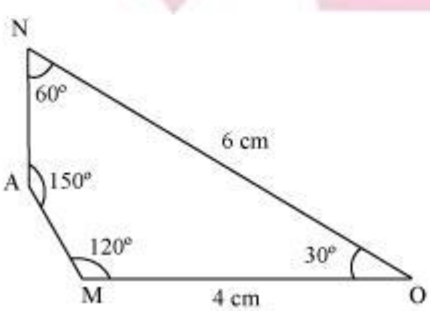
∴ ∠M + ∠O + ∠N + ∠A = 360°

⇒ 120° + ∠O + 60° + 150° = 360°

⇒ ∠O = 30°

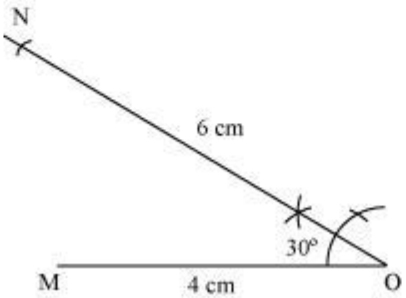
Step 1:

A rough figure of this quadrilateral is drawn first.



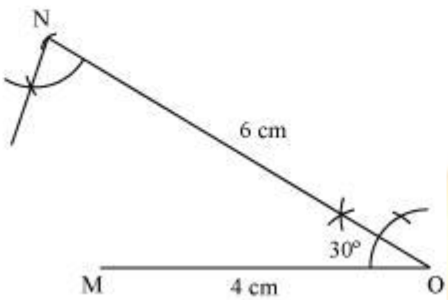
Step 2:

Draw a line segment MO of 4 cm and form a 30° angle at point O. Taking O as the centre, draw an arc of 6 cm which cuts this ray at point N.



Step 3:

Draw a ray at point N making an angle of 60° with line segment NO.

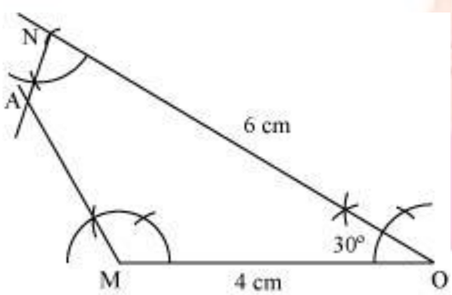


Step 4:

Draw a ray at point M making an angle of 120° with line segment MO.

Let these two rays intersect at point A.

Thus, MONA is the required quadrilateral.



Question 17 (3.0 marks)
Use the following information to answer the next question.

$P = ab$ is a two-digit number and Q is the number formed by reversing the digits of P i.e., ba.

I. $P + Q$ is a prime number.

II. $P - Q$ is a multiple of 3 and 9 both.

Are these statements always correct? Give reasons.

Solution:
$P = ab = 10a + b$

$Q = ba = 10b + a$

I. $P + Q = (10a + b) + (10b + a) = 11a + 11b = 11\ (a + b)$

It can be seen that $P + Q$ will always be a multiple of 11 and the sum of its digits, i.e. $a + b$, in each case. Thus, it cannot be a prime number.

II. $P - Q = (10a + b) - (10b + a) = 9a - 9b = 9\ (a - b)$

It means that either $a > b$ or $a < b$. In each case, $P - Q$ will be a multiple of 9. If a number is a multiple of 9, then it will be a multiple of 3 also.

Question 18 (3.0 marks)
Use the following information to answer the next question.

By using 3 different non-zero digits, J, K, and L, four numbers are formed such that

$A = J\ K\ L$

$B = L\ J\ K$

$C = K\ L\ J$

$D = L\ K\ J$

(i) Write four factors other than 1 for the expression $A - D$.

(ii) Is the expression $(A + B + C)$ a prime number? Give reasons.

Solution:
$A = J\ K\ L = 100\ J + 10\ K + L$

$B = L\ J\ K = 100\ L + 10\ J + K$

$C = K\ L\ J = 100\ K + 10\ L + J$

$D = L\ K\ J = 100\ L + 10\ K + J$

(i) $A - D = (100\ J + 10\ K + L) - (100\ L + 10\ K + J) = 99\ J - 99\ L = 99\ (J - L)$

Expression $A - D$ is a multiple of 99. Its factors are 3, 9, 11, and 33. Thus, the expression $A - D$ is also a multiple of 3, 9, 11, and 33.

(ii) $A + B + C = (100\,J + 10\,K + L) + (100\,L + 10\,J + K) + (100\,K + 10\,L + J) = 111\,(J + K + L)$

It can be seen that $111 = 37 \times 3$. Therefore, the expression $A + B + C$ is not a prime number.

Question 19 (3.0 marks)
(a) There are 96 non-perfect square numbers between the squares of two natural numbers. Find these numbers.

(b) Find the square root of the sum of all natural numbers that are odd and less than 1000.

(c) If $486 \times 488 = a^2 - 1$, then find the value of a.

Solution:
(a)We know that $2n$ non-perfect square numbers lie between the squares of two numbers n and $n + 1$.

Therefore, 96 non-perfect square numbers will lie between the squares of the numbers 48 and 49.

(b)We know that the sum of first n odd numbers $(1 + 3 + 5 + 7 + 9\ldots n)$ is n^2.

We have to find $1 + 3 + 5 + 7 + \ldots + 999$.

There are 500 odd numbers between 0 and 1000.

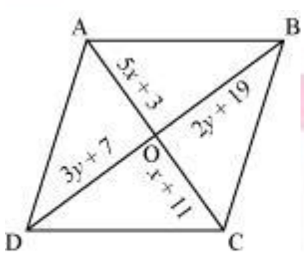
$\therefore\ 1 + 3 + 5 + 7 + \ldots + 999 = (500)^2$

Thus, the square root of this sum is 500.

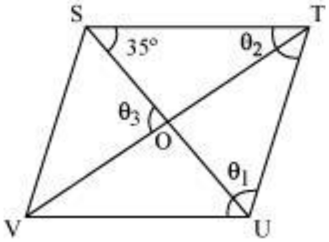
(c)$486 \times 488 = (487 - 1)\,(487 + 1) = (487)^2 - 1$

Thus, the value of a is 487.

Question 20 (3.0 marks)
(i) Find the area of the given rhombus.



(ii) The given figure shows a rhombus STUV. Find the value of expression $(\theta_2 + \theta_3 - 2\theta_1)$.



Solution:

(i) We know that in rhombus, the diagonals bisect each other.

$\therefore 3y + 7 = 2y + 19$

$\Rightarrow y = 12$

$5x + 3 = x + 11$

$\Rightarrow x = 2$

$\therefore$ AO = OC = $5x + 3 = 5\,(2) + 3 = 13$ units

AC = 2AO = 26 units

BO = DO = $2y + 19 = 2(12) + 19 = 43$ units

BD = 2 (43) = 86 units

$\therefore$ Area of rhombus ABCD = $\frac{1}{2} \times \text{AC} \times \text{BD}$

$= \left(\frac{1}{2} \times 26 \times 86\right)$ sq. units

$= 1118$ sq. units

(ii) We know that the diagonals of a rhombus bisect each other at 90°. Also, the diagonals are angle bisectors.

$\therefore \theta_3 = \angle SOV = 90°$

$\Rightarrow \theta_1 = 2\angle OST = 2\,(35°) = 70°$

$\theta_2 + \theta_1 = 180°$

(Interior angles on the same side of transversal are supplementary)

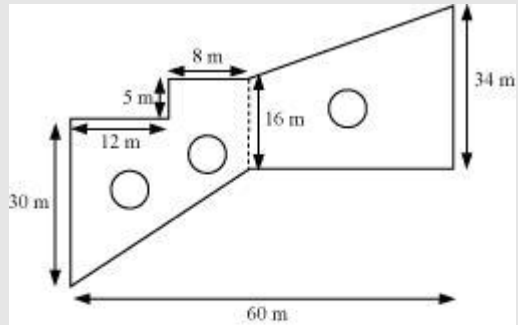
$\Rightarrow \theta_2 = 180° - 70° = 110°$

$\therefore \theta_2 + \theta_3 - 2\theta_1 = 110° + 90° - 2\,(70°) = 200° - 140° = 60°$

Thus, the value of expression $(\theta_2 + \theta_3 - 2\theta_1)$ is 60°.

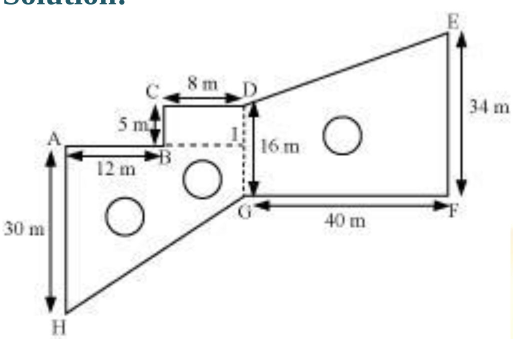
Question 21 (3.0 marks)
Use the following information to answer the next question.

The given figure shows the layout of a park to be constructed with three circular ponds each of 7 m radius in it. The rest of the area, other than ponds, is to be covered with grass.



If the cost of covering the park with grass is Rs 2/m^2, then find the total cost of covering the park with grass.

Solution:



It can be seen that,

Area of trapezium AIGH = $\frac{1}{2}\times(\text{AH}+\text{IG})\times\text{AI}$

$=\left(\frac{1}{2}\times(30+11)\times 20\right)\ \text{m}^2$

$=(41\times 10)\ \text{m}^2 = 410\ \text{m}^2$

Area of rectangle BIDC = CD × CB = (8 × 5) m^2 = 40 m^2

Area of trapezium GFED = $\frac{1}{2}\times(\text{DG}+\text{EF})\times\text{GF}$

$=\left(\frac{1}{2}\times(16+34)\times 40\right)\ \text{m}^2$

$=1000\ \text{m}^2$

∴ Area of park = (410 + 40 + 1000) m^2 = 1450 m^2

Area of circular pond = $\pi r^2 = \frac{22}{7}\times(7\ \text{m})^2 = 154\ \text{m}^2$

⇒ Area of three circular ponds = 3 × 154 m^2 = 462 m^2

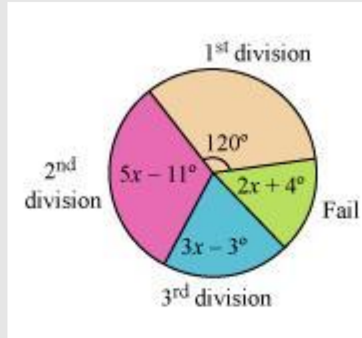
∴ Area of park which is to be covered with grass = (1450 − 462) m^2 = 988 m^2

Cost of covering 1 m^2 area with grass = Rs 2

∴ Cost of covering 988 m^2 area with grass = Rs 2 × 988 = Rs 1976

Thus, the total cost of covering the park with grass is Rs 1976.

Question 22 (3.0 marks)
Use the following information to answer the next question.

The given pie-chart represents the result of an examination of class VIII of a school. 90 students failed in the examination.	

I. How many students appeared in the exam?

II.What percentage of students passed the exam?

III.What is the difference between the number of students getting first division and third division?

Solution:
In the given pie-chart,

$120° + 5x - 11° + 3x - 3° + 2x + 4° = 360°$

$\Rightarrow$ $10x + 110° = 360°$

$\Rightarrow$ $10x = 250°$

$\Rightarrow$ $x = 25°$

I. Central angle of failed students = $2x + 4° = 2 \times 25° + 4° = 54°$

54° of pie-chart represents 90 students.

∴ 360° of pie-chart represents, $\frac{90 \times 360°}{54°}$ = 600 students

Therefore, 600 students appeared for the exam.

II. Central angle of passed students = 360° − 54° = 306°

Percentage of passed students in the given pie-chart, $\left(\frac{306°}{360°}\times 100\right)\% = 85\%$

III. Central angle of students getting first division = 120°

Central angle of students getting third division = $3x - 3°$

$= 3 \times 25° - 3°$

$= 72°$

Difference between the central angles of first division and third division students = (120° − 72°) = 48°

Total number of students that appeared for exam = 600

Therefore, required difference between the number of students getting first division and third division = $\frac{48°}{360°}\times 600 = 80$

Question 23 (3.0 marks)
A car was bought at Rs 300000. Its value depreciated at the rate of 7% per annum. Find its value after 2 years.

Solution:
P = Rs 300000

$r = 7\%$

$n = 2$

$$\begin{aligned} A &= P\left(1-\frac{r}{100}\right)^2 \\ &= 300000\left(1-\frac{7}{100}\right)^2 \\ &= 300000\left(\frac{93}{100}\right)^2 \\ &= 30\times 93\times 93 \\ &= 259470 \end{aligned}$$

Thus, the value of the car will be Rs 259470 after 2 years.

Question 24 (3.0 marks)
25 men were assigned to do a work in 40 days. However, due to some reasons, 5 men did not turn up for the work. How many more days are required by the remaining men to complete the work?

Solution:
Let the number of days required to complete the work be x.

It can be seen that lesser the number of workers, more will be the time taken by them to complete the work. Therefore, it is a case of inverse proportion.

Number of workers	25	20
Days	40	x

$\therefore 25 \times 40 = 20 \times x$

$\Rightarrow x = 50$

Thus, (50 − 40) = 10 more days are required to complete the work.

Question 25 (3.0 marks)
Factorise the expression $9m^3 - 27m^2 - 16m + 48$.

Solution:

$$9m^3 - 27m^2 - 16m + 48$$
$$= 9m^3 - 16m - 27m^2 + 48$$
$$= m\left(9m^2 - 16\right) - 3\left(9m^2 - 16\right)$$
$$= \left(9m^2 - 16\right)(m - 3)$$
$$= \left\{(3m)^2 - (4)^2\right\}(m - 3)$$
$$= (3m - 4)(3m + 4)(m - 3)$$

Question 26 (6.0 marks)
(a) *Use the following information to answer the next question.*

$$\begin{array}{r} 3B \\ \times \quad 7 \\ \hline \underline{2CC} \end{array}$$

If B and C represent a single digit number and $B \neq C$, then find the values of B and C.

(b) *Use the following information to answer the next question.*

$$\begin{array}{r} BCD \\ +\ CC5 \\ \hline \underline{A0BA} \end{array}$$

If A, B, C, D are single digit numbers, not equal to each other, and are other than 0 and 5, then find the values of A, B, C, and D.

Solution:
(a)

$$\begin{array}{r} 3B \\ \times\ \ 7 \\ \hline 2CC \\ \hline \end{array}$$

It can be seen that in the given problem, $3 \times 7 = 2C$. Therefore,

(i) $C = 1 + 0 = 1$. It is possible only if $B = 1$.

$\Rightarrow B = C$, which is not possible.

(ii) $C = 1 + 1 = 2$. It is possible only if $B = 2$.

$\Rightarrow B = C$, which is not possible.

(iii) $C = 1 + 2 = 3$. It is possible only if $B = 3$ or 4.

If $B = 3$, then $B = C$, which is not possible.

If $B = 4$, then the product will be 238, which is not of type $2CC$.

(iv) $C = 1 + 3 = 4$. It is possible only if $B = 5$.

Then, the product will be 245, which is not of type $2CC$.

(v) $C = 1 + 4 = 5$. It is possible only if $B = 6$ or 7.

If $B = 6$, then the product will be 252, which is not of type $2CC$.

If $B = 7$, then the product will be 259, which is not of type $2CC$.

(vi) $C = 1 + 5 = 6$. It is possible only if $B = 8$.

If $B = 8$, then the product will be 266, which is of the type $2CC$.

Thus, the value of B and C is 8 and 6 respectively.

The given multiplication becomes

$$\begin{array}{r} 38 \\ \times\ \ 7 \\ \hline 266 \\ \hline \end{array}$$

(b)

$$\begin{array}{r} BCD \\ +\ CC5 \\ \hline A0BA \\ \hline \end{array}$$

It can be seen that B and C are different digits. Therefore, the maximum value of $B + C$ can be 17 only.

Therefore, A can be 1 only.

We obtain,

$$\begin{array}{r} BCD \\ +\ CC5 \\ \hline 10B1 \\ \hline \end{array}$$

$D + 5 = 1$

It is possible only if D is 6.

It means that 1 is carried in the next step. Therefore, we obtain

$1 + C + C = B$

$1 + 2C = B$

Also, $B + C = 10$

$\Rightarrow 1 + 2C + C = 10$

$\Rightarrow 1 + 3C = 10$

$\therefore C = 3$

$\Rightarrow B = 10 - 3 = 7$

Thus, the values of A, B, C, and D are 1, 7, 3, and 6 respectively.

The given addition becomes

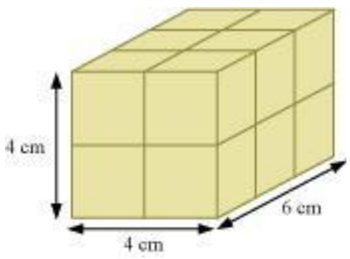
$$\begin{array}{r} 736 \\ +\ 335 \\ \hline 1071 \\ \hline \end{array}$$

Question 27 (6.0 marks)
(a) A cuboid of dimension 6 cm × 4 cm × 4 cm is cut into cubes of side 2 cm. Find the percentage increase in the surface area of the cuboid.

(b)A rectangle sheet of length 50 cm and width 44 cm is rolled along its width. Find the volume of the cylinder so formed. $\left(\text{Assume } \pi = \frac{22}{7}\right)$

(c)A cubical tank of side 6 m is fully filled with water. A hole in the bottom of the tank can empty it in 60 hours. Find the rate of flow of water through this hole.

Solution:
(a)



It can be seen from the figure that 12 cubes of size 2 cm can be cut from the given cuboid.

Total surface area of original cuboid = 2 ($l \times b + b \times h + l \times h$)

$= 2\,(6 \times 4 + 4 \times 4 + 6 \times 4)\ \text{cm}^2$

$= 2\,(24 + 16 + 24)\ \text{cm}^2$

$= 128\ \text{cm}^2$

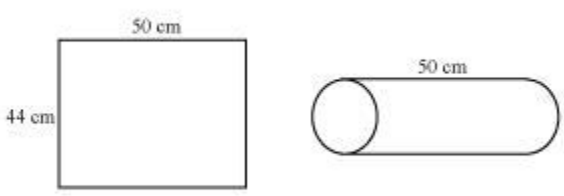
Surface area of cube = $6 \times (2\ \text{cm})^2 = 24\ \text{cm}^2$

Surface area of 12 cubes = $12 \times 24\ \text{cm}^2 = 288\ \text{cm}^2$

$\therefore$ Percentage increase in total surface area = $\frac{(288-128)\ \text{cm}^2}{128\ \text{cm}^2} \times 100 = 125\%$

Thus, the percentage increase in the surface area of cuboid is 125%.

(b)



When a rectangular sheet is folded along its width, its length becomes the height of the cylinder and width becomes the perimeter of the base of the cylinder.

Let the radius of the base of the cylinder be *r*.

$\therefore 2\pi r = 44\ cm^2$

$$2 \times \frac{22}{7} \times r = 44\ cm^2$$
$$r = 7\ cm$$

Volume of cylinder = $\pi r^2 h$

$$= \frac{22}{7} \times (7\ cm)^2 \times 50\ cm$$
$$= 7700\ cm^3$$
$$= 7.7\ L$$

(c) Volume of cubical tank = $(6\ m)^3 = 216\ m^3$

Volume of water in cubical tank = $216\ m^3$ = 216000 L

$\therefore$ Speed of flow of water = $\frac{\text{Volume of water}}{\text{Time taken to empty the tank}}$

$$= \frac{216000}{60} L/h$$

= 3600 L/h

= 60 L/min

Question 28 (6.0 marks)
Use the following information to answer the next question.

A financial company offers 20% simple interest (S.I.) on deposits under its special scheme.

Draw a graph to illustrate the relation between the sum deposited and simple interest earned. Then from the graph, find

(a)the annual interest obtainable on deposit of Rs 4500

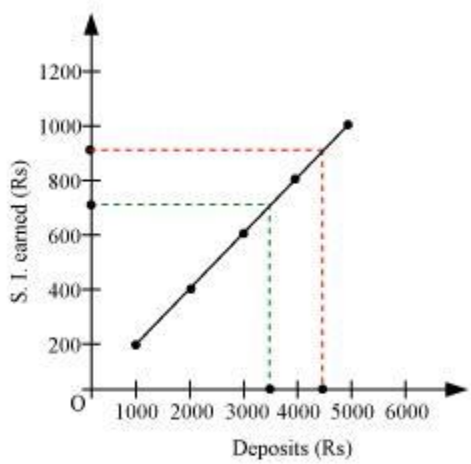
(b)the investment one has to make to obtain an annual simple interest of Rs 700

Solution:
The simple interest earned on different sums is calculated as follows.

Sum deposited (Rs)	**Simple interest for a year (Rs)**

1000	$\frac{1000 \times 20 \times 1}{100} = 200$
2000	$\frac{2000 \times 20 \times 1}{100} = 400$
3000	$\frac{3000 \times 20 \times 1}{100} = 600$
4000	$\frac{4000 \times 20 \times 1}{100} = 800$
5000	$\frac{5000 \times 20 \times 1}{100} = 1000$

Taking simple interest earned along vertical axis, deposits along horizontal axis, and scale as 1 unit = Rs 200 along vertical axis and 1 unit = Rs 1000 along horizontal axis. Plotting these points and joining them by line segments, we obtain the graph as



(a) It can be seen from the graph that on a deposit of Rs 4500, the simple interest earned is Rs 900.

(b) Also, a simple interest of Rs 700 is obtained on deposit of Rs 3500.

Question 29 (6.0 marks)
(a) An article is sold at 15% discount. The shopkeeper marked up the price of this article by 25%.

Find the profit or loss percentage incurred by this shopkeeper.

(b)Find the compound interest earned on a sum of Rs 10000 in 1.5 years at a rate of 10% per annum compounded half yearly.

Solution:
(a)Let the cost price of the article be Rs 100.

∴ Marked price = $\text{Rs } 100+\frac{25}{100}\times \text{Rs } 100$

If marked price is Rs 100, then discount = Rs 15

If marked price is Re 1, then discount = $\text{Rs}\frac{15}{100}$

If marked price is Rs 125, then discount $=\text{Rs}\left(\frac{15}{100}\times 125\right)=\text{Rs }\frac{75}{4}=\text{Rs } 18.75$

∴ Selling price = Marked price − Discount = Rs 125 − Rs 18.75 = Rs 106.25

It can be seen that the shopkeeper sold the article of Rs 100 for Rs 106.25. Therefore, he earned a profit of Rs 6.25.

Thus, the profit percentage incurred by the shopkeeper is 6.25%.

(b) We know that, 1.5 years = 3 half years

Since the interest is compounded half yearly, the rate of interest (r) = 5%

$$\begin{aligned}A &= \text{P}\left(1+\frac{r}{100}\right)^n\\ &= \text{Rs } 10000\left(1+\frac{5}{100}\right)^3\\ &= \text{Rs } 10000\left(1+\frac{1}{20}\right)^3\\ &= \text{Rs } 10000\left(\frac{21}{20}\right)^3\\ &= \text{Rs } 10000\times 1.157625\\ &= \text{Rs } 11576.25\end{aligned}$$

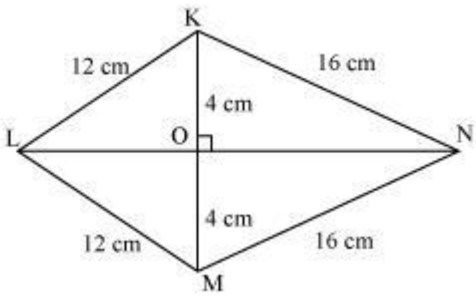
∴ Compound interest = Rs 11576.25 − Rs 10000 = Rs 1576.25

Thus, the compound interest is Rs 1576.25.

Question 30 (6.0 marks)
Construct a kite-shaped quadrilateral KLMN, where equal pair of adjacent sides measure 12 cm (KL and LM) and 16 cm (KN and MN) and the smaller diagonal measures 8 cm.

Solution:
Step 1:

A rough sketch of this quadrilateralis drawn as follows.



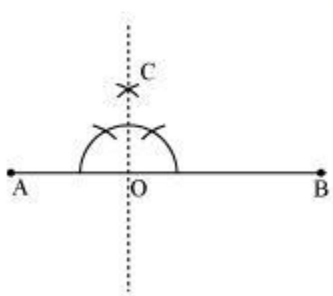
Step 2:

Draw a line $\overleftrightarrow{AB}$ of a convenient length.

Step 3:

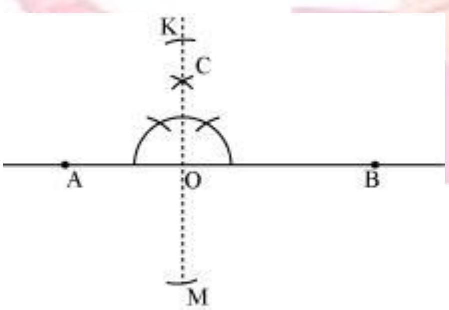
Then, draw a ray OC making a 90° angle at any point O of this line $\overleftrightarrow{AB}$.

Also, expand it downwards.



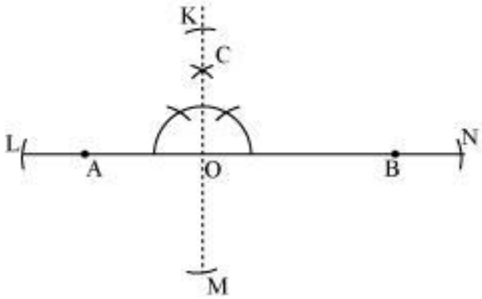
Step 4:

We know that the bigger diagonal of kite is the perpendicular bisector of the smaller one. Hence, KO and MO will measure 4 cm each. Therefore, draw arcs of 4 cm on this ray while taking O as centre.



Step 5:

Then, draw arcs of 12 cm and 16 cm radius while taking K as centre to cut $\overleftrightarrow{AB}$ at L and N respectively.



Step 6:

Join KL, KN, ML, and MN. KLMN is the required kite.