Test 1

Subjective Test

(i) All questions are compulsory.

(ii) Questions 1 to 8 are multiple choice questions carrying one mark each.

(iii) Questions 9 to 12 are also multiple choice questions carrying two marks each.

(iv) Questions 13 to 19 are short answer type questions carrying two marks each.

(v) Questions 20 to 29 are also short answer type questions carrying three marks each.

(vi) Questions 30 to 34 are long answer type questions carrying four marks each.

Question 1 (1.0 marks)
Which of the following numbers is irrational?

A.

0.0173101731…

B.

0.0769207692…

C.

0.03899721448…

D.

0.50349603496…

Solution:
A number whose decimal expansion is terminating or non-terminating recurring is rational. On the other hand, a number whose decimal expansion is non-terminating and non-recurring is irrational.

It is seen that the number 0.03899721448… has a non-terminating and a non-recurring decimal expansion.

Thus, 0.03899721448… is an irrational number.

The correct answer is C.

Why alternative A is wrong:

The number 0.0173101731… $= 0.\overline{01731}$ has a non-terminating but recurring decimal expansion. Thus, it is a rational number.

Why alternative B is wrong:

The number 0.0769207692… $= 0.\overline{07692}$ has a non-terminating but recurring decimal expansion. Thus, it is a rational number.

Why alternative D is wrong:

The number 0.50349603496… $= 0.5\overline{03496}$ has a non-terminating but recurring decimal expansion. Thus, it is a rational number.

Question 2 (1.0 marks)

What is the remainder when the polynomial, $x^3 + 7x^2 + 18x + 25$, is divided by another polynomial $(x+4)$?

A.

1

B.

2

C.

4

D.

5

Solution:
According to Remainder Theorem, when a polynomial *p*(*x*) is divided by a linear polynomial (*x* − *a*), the remainder obtained is*p*(*a*).

Here, *p*(*x*) =, $x^3 + 7x^2 + 18x + 25$ which is divided by $(x+4)$

The zero of the linear polynomial $(x+4)$ is −4.

∴ Remainder obtained = *p*(−4)

$$\begin{aligned} p(-4) &= (-4)^3 + 7(-4)^2 + 18(-4) + 25 \\ &= -64 + 7(16) - 72 + 25 \\ &= -64 + 112 - 72 + 25 \\ &= 1 \end{aligned}$$

Thus, when the polynomial, $x^3 + 7x^2 + 18x + 25$ is divided by another polynomial $(x+4)$, the remainder obtained is 1.

The correct answer is A.

Question 3 (1.0 marks)
What is the zero of the polynomial $p(x) = 3x + 1$?

A.

-2

B.

$-\frac{1}{3}$

C.

0

D.

$\frac{1}{3}$

Solution:
The zero of the polynomial $p(x)$ is given by $p(x) = 0$

$$\Rightarrow 3x + 1 = 0$$
$$\Rightarrow 3x = -1$$
$$\Rightarrow x = \left(-\frac{1}{3}\right)$$

Thus, $\left(-\frac{1}{3}\right)$ is the zero of the given polynomial.

The correct answer is B.

Question 4 (1.0 marks)

How can the expression $\frac{4}{3-\sqrt{5}}$ be simplified?

A.

$3+\sqrt{5}$

B.

$4\left(3+\sqrt{5}\right)$

C.

$3-\sqrt{5}$

D.

$2\left(3-\sqrt{5}\right)$

Solution:

$$\frac{4}{3-\sqrt{5}}$$

$$=\frac{4}{3-\sqrt{5}}\times\frac{3+\sqrt{5}}{3+\sqrt{5}} \qquad \left[\text{Multiplying numerator and denominator by } 3+\sqrt{5}\right]$$

$$=\frac{4\left(3+\sqrt{5}\right)}{(3)^2-\left(\sqrt{5}\right)^2} \qquad \left[a^2-b^2=(a+b)(a-b)\right]$$

$$=\frac{4\left(3+\sqrt{5}\right)}{9-5}$$

$$=\frac{4\left(3+\sqrt{5}\right)}{4}$$

$$=3+\sqrt{5}$$

The correct answer is A.

Question 5 (1.0 marks)

How can the expression $\left(\frac{x^a}{x^b}\right)^c\left(\frac{x^b}{x^c}\right)^a\left(\frac{x^c}{x^a}\right)^b$ be simplified?

A.

1

B.

2

C.

x^{abc}

D.

$x^{(a-b)c}$

Solution:

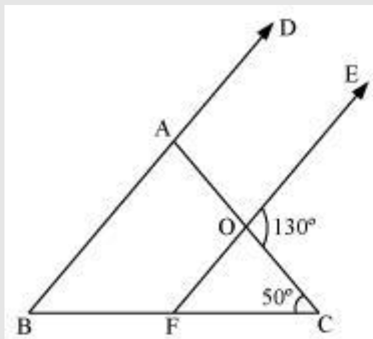
$$\left(\frac{x^a}{x^b}\right)^c\left(\frac{x^b}{x^c}\right)^a\left(\frac{x^c}{x^a}\right)^b$$

$$=\frac{x^{ac}}{x^{bc}}\times\frac{x^{ab}}{x^{ac}}\times\frac{x^{bc}}{x^{ab}} \qquad \left[\left(a^m\right)^n=a^{mn}\right]$$

$$=1$$

The correct answer is A.

Question 6 (1.0 marks)
Use the following information to answer the next question.

In the given figure, BD||EF.	

What is the measure of ∠ABC in the given figure?

A.

55°

B.

65°

C.

70°

D.

80°

Solution:
In ΔOFC, ∠EOC = ∠OCF + ∠OFC

∴ ∠OFC = 130° − 50° = 80°

BC is parallel to EF.

∴ ∠ABC = ∠OFC (Corresponding angles)

⇒ ∠ABC = 80°

Thus, the measure of ∠ABC is 80°.

The correct answer is D.

Question 7 (1.0 marks)
Use the following information to answer the next question.

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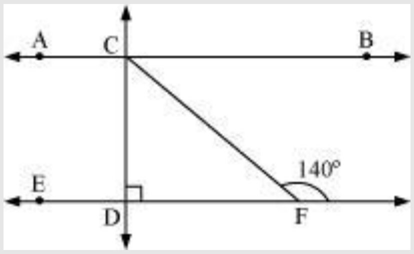


If AB||EF, then what is the measure of ∠DCF?

A.

30°

B.

40°

C.

50°

D.

60°

Solution:
It is given that AB||EF.

∴∠ACD = ∠CDF (Pair of alternate interior angles)

⇒ ∠ACD = 90° (∠CDF = 90°)

∠ACF = 140° (Pair of alternate interior angles)

⇒∠ACD + ∠DCF = 140°

⇒ 90° + ∠DCF = 140°

⇒ ∠DCF = 140° − 90° = 50°

Thus, the measure of ∠DCF is 50°.

The correct answer is C.

Question 8 (1.0 marks)
Which of the following statements is **not** an equivalent version of Euclid's fifth postulate?

A.

Two distinct intersecting lines cannot be parallel to the same line.

B.

For every line *l* and for every point T not lying on *l*, their exists a unique line *m* passing through T and parallel to *l*.

C.

If a straight line crossing two straight lines makes interior angles measuring less than two right angles on the same side, then the two lines, if extended indefinitely, will meet on that side on which the angles measure more than two right angles.

D.

If two parallel lines are cut by a transversal, then the alternate interior angles are equal and the corresponding angles are also equal.

Solution:
An equivalent version of Euclid's fifth postulate can be written as:

If a straight line crossing two straight lines makes interior angles measuring less than two right angles on the same side, then the two lines, if extended indefinitely, will meet on that side on which the angles measure less than two right angles.

Thus, the statement given in alternative **C** is **not** an equivalent version of Euclid's fifth postulate.

The correct answer is C.

Question 9 (2.0 marks)
Which rational number is equal to the decimal $0.34\overline{89}$?

A.

$\frac{69}{180}$

B.

$\frac{87}{250}$

C.

$\frac{349}{1000}$

D.

$\frac{691}{1980}$

Solution:
Let x = $0.34\overline{89}$ = 0.34898989…

⇒ 100x = 34.898989… … (1)

⇒ 10000x = 3489.8989… … (2)

Subtracting (1) from (2), we obtain

10000x − 100x = 3489.8989… − 34.898989…

9900x = 3455

$\Rightarrow x = \frac{3455}{9900} = \frac{691}{1980}$

Thus, $0.34\overline{89}$ can be written in $\frac{p}{q}$ form as $\frac{691}{1980}$.

The correct answer is D.

Question 10 (2.0 marks)
If in ΔPQR, ∠P = 70° and ∠Q = 75°, then which of the following relations is correct?

A.

PQ > QR > PR

B.

PQ > PR > QR

C.

QR > PR > PQ

D.

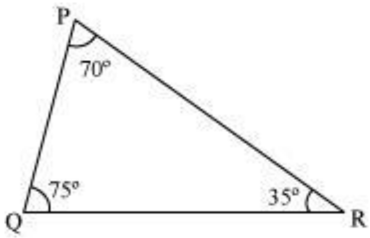
PR > QR > PQ

Solution:
Using angle sum property for ΔPQR, we obtain

$\angle P + \angle Q + \angle R = 180°$

$\Rightarrow 70° + 75° + \angle R = 180°$

$\Rightarrow \angle R = 180° - 145° = 35°$



In ΔPQR,

$\angle P = 70°$, $\angle Q = 75°$, and $\angle R = 35°$

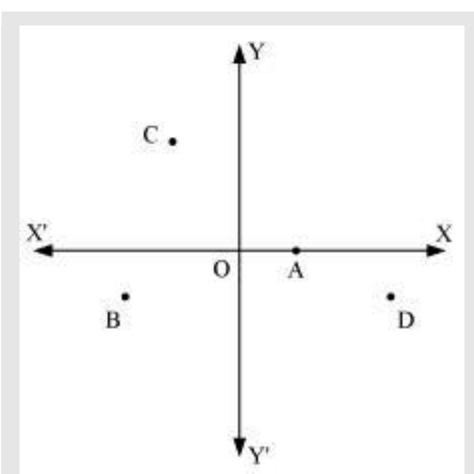
$\therefore\ \angle Q > \angle P$ and $\angle P > \angle R$

$\Rightarrow PR > QR$ and $QR > PQ$ (Sides opposite to greater angles are longer)

$\Rightarrow PR > QR > PQ$

The correct answer is D.

Question 11 (2.0 marks)
Use the following information to answer the next question.



Among the given points **A**, **B**, **C** and **D**; which point has negative abscissa and positive ordinate?

A.

A

B.

B

C.

C

D.

D

Solution:
Point **A** has positive abscissa.

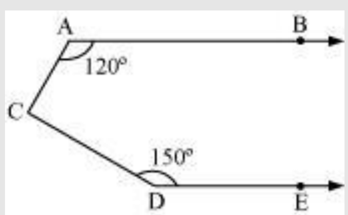
Points **B** and **C** have negative abscissa.

Point **C** has positive ordinate.

Thus, the point **C** has negative abscissa and positive ordinate.

The correct answer is C.

Question 12 (2.0 marks)
Use the following information to answer the next question.

In the given figure, AB||DE.	

What is the measure of ∠ACD?

A.

60°

B.

90°

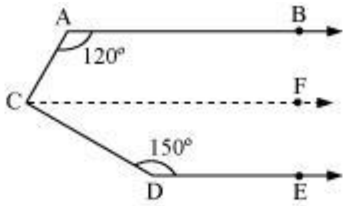
C.

110°

D.

120°

Solution:
Draw CF||AB||DE.



It is known that the sum of adjacent interior angles on the same side of transversal is 180°.

∴ ∠FCD + ∠CDE = 180°

⇒ ∠FCD + 150° = 180°

⇒ ∠FCD = 180° − 150° = 30°

Similarly, ∠ACF = 180° − 120° = 60°

∴ ∠ACD = ∠FCD + ∠ACF = 30° + 60° = 90°

The correct answer is B.

Question 13 (2.0 marks)

Find a rational number which is equal to the expression, $2.3\overline{6} \div 1.\overline{5}$.

Solution:
The given expression is $2.3\overline{6} \div 1.\overline{5}$.

Let $x = 2.3\overline{6}$ … (1), $y = 1.\overline{5}$ … (2)

Multiplying equation (1) by 10, we obtain

$10x = 23.\overline{6}$ … (3)

Subtracting equation (1) from equation (3), we obtain

$$10x - x = 23.\overline{6} - 2.3\overline{6}$$
$$\Rightarrow 9x = 21.3$$
$$\Rightarrow x = \frac{21.3}{9} = \frac{213}{90}$$

On multiplying equation (2) by 10, we obtain

$10y = 15.\overline{5}$ … (5)

Subtracting equation (2) from equation (5), we obtain

$$10y - y = 15.\bar{5} - 1.\bar{5}$$
$$\Rightarrow 9y = 15.\bar{5} - 1.\bar{5} = 14$$
$$\Rightarrow y = \frac{14}{9}$$

Thus, $2.3\bar{6} \div 1.\bar{5} = \frac{213}{90} \div \frac{14}{9} = \frac{213}{90} \times \frac{9}{14} = \frac{213}{140}$

Question 14 (2.0 marks)

Simplify the expression $\left(\left(\sqrt[3]{3}\right)^{\sqrt[3]{3}}\right)^{\sqrt[3]{9}}$.

Solution:
The given expression can be simplified as:

$$\left(\left(\sqrt[3]{3}\right)^{\sqrt[3]{3}}\right)^{\sqrt[3]{9}} = \left(\sqrt[3]{3}\right)^{\sqrt[3]{3}\times\sqrt[3]{9}} \qquad \left[\left(a^m\right)^n = a^{mn}\right]$$
$$= \left(\sqrt[3]{3}\right)^{3^{\frac{1}{3}}\times 3^{\frac{2}{3}}} \qquad \left[\sqrt[3]{9} = \left(3^2\right)^{\frac{1}{3}} = 3^{\frac{2}{3}}\right]$$
$$= \left(\sqrt[3]{3}\right)^{3^{\frac{1}{3}+\frac{2}{3}}} \qquad \left[a^m \times a^n = a^{m+n}\right]$$
$$= \left(\sqrt[3]{3}\right)^{3^1}$$
$$= \left(3^{\frac{1}{3}}\right)^3 = 3^{\frac{1}{3}\times 3} = 3$$

Thus, the value of the expression $\left(\left(\sqrt[3]{3}\right)^{\sqrt[3]{3}}\right)^{\sqrt[3]{9}}$ is 3.

Question 15 (2.0 marks)
The polynomial $p(x) = x^3 + ax^2 - 11x - 12$ is exactly divisible by $(x + 1)$. Find the value of a.

Solution:
The given polynomial is $p(x) = x^3 + ax^2 - 11x - 12$

By Factor theorem, it is known that if $x - a$ is a factor of polynomial $p(x)$, then $p(a) = 0$

It is given that $p(x) = x^3 + ax^2 - 11x - 12$ is exactly divisible by $(x + 1)$. Therefore, $(x + 1)$ is a factor of $p(x)$.

Therefore, we must have $p(-1) = 0$

$\therefore\ p(-1) = 0$

$\Rightarrow (-1)^3 + a(-1)^2 - 11(-1) - 12 = 0$

$\Rightarrow -1 + a + 11 - 12 = 0$

$\Rightarrow -13 + a + 11 = 0$

$\Rightarrow a = 13 - 11 = 2$

Thus, the required value of a is 2.

Question 16 (2.0 marks)
If $(x - 1)$ is a factor of $p(x) = kx^2 + (2k + 1)x - 13$, then find the value of k. Factorise $p(x)$.

Solution:
$(x - 1)$ is a factor of $p(x) = kx^2 + (2k + 1)x - 13$. Therefore, by factor theorem, we obtain

$p(1) = 0$

$k(1)^2 + (2k + 1)(1) - 13 = 0$

$k + (2k + 1) - 13 = 0$

$k = 4$

$\therefore\ p(x) = 4x^2 + 9x - 13$

Splitting the middle term, we obtain

$p(x) = 4x^2 + 13x - 4x - 13$

$= x(4x + 13) - 1(4x + 13)$

$= (x - 1)(4x + 13)$

Thus, $p(x)$ can be factorized as $(x - 1)(4x + 13)$.

Question 17 (2.0 marks)
Two polynomials, $x^3 - 5x^2 + ax - 1$ and $2x^3 + x^2 - (a + 1)x + 1$, when divided by

$(x - 1)$ gives remainder k_1 and k_2. If $2k_1 + k_2 = 2$, then find the value of a.

Solution:
We know by remainder theorem that when $p(x)$ is divided by $(x - b)$, the remainder is $p(b)$.

Let $p_1(x) = x^3 - 5x^2 + ax - 1$

$p_2(x) = 2x^3 + x^2 - (a + 1)x + 1$

$\therefore\ p_1(1) = (1)^3 - 5(1)^2 + a(1) - 1 = k_1$

$\Rightarrow k_1 = -5 + a$

$\therefore\ p_2(1) = 2(1)^3 + (1)^2 - (a + 1)(1) + 1 = k_2$

$\Rightarrow k_2 = 3 - a$

Also, $2k_1 + k_2 = 2$

$\therefore\ 2(-5 + a) + (3 - a) = 2$

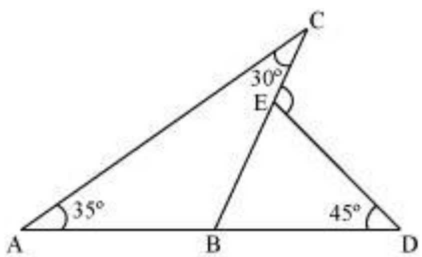
$\Rightarrow -10 + 2a + 3 - a = 2$

$\Rightarrow a - 7 = 2$

$\therefore\ a = 9$

Thus, the value of a is 9.

Question 18 (2.0 marks)
What is the measure of ∠CED in the given figure?



Solution:
Applying exterior angle property in ΔABC:
∠CBD = ∠BAC + ∠ACB = 35° + 30° = 65° … (1)

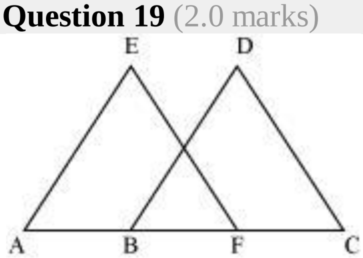
Similarly, applying exterior angle property in ΔBDE:

∠CED = ∠EBD + ∠BDE

= 65° + 45° (Using (1))

= 110°

Question 19 (2.0 marks)



In the given figure, AB = CF, EF = BD and ∠EFC = ∠ABD. Prove that ΔAFE ≅ ΔCBD.

Solution:
It is given that AB = CF

$\therefore$ AB + BF = CF + BF

⇒ AF = BC … (1)

It is also given that ∠EFC = ∠ABD

∴ 180° − ∠AFE = 180° − ∠CBD

⇒ ∠AFE = ∠CBD … (2)

Comparing ΔAFE and ΔCBD:

AF = BC [Using (1)]

∠AFE = ∠CBD [Using (2)]

EF = BD [Given]

∴ ΔAFE ≅ ΔCBD [SAS congruency criterion]

Question 20 (3.0 marks)

Divide $\left(x^3 - \frac{15}{2}x^2 + 29x - 6\right)$ by (x − 2).

Solution:

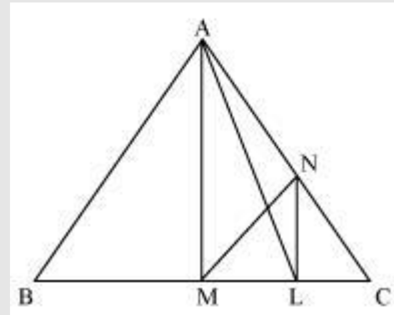
The polynomial $\left(x^3 - \frac{15}{2}x^2 + 29x - 6\right)$ can be divided by the polynomial (x − 2) as

$$
\begin{array}{r l}
 & x^2 - \frac{11}{2}x + 18 \\
\cline{2-2}
(x-2) & \big) \; x^3 - \frac{15}{2}x^2 + 29x - 6 \\
 & \;\; x^3 - 2x^2 \\
 & \;\; - \quad + \\
\cline{2-2}
 & \qquad -\frac{11}{2}x^2 + 29x - 6 \\
 & \qquad -\frac{11}{2}x^2 + 11x \\
 & \qquad + \qquad\quad - \\
\cline{2-2}
 & \qquad\qquad\qquad 18x - 6 \\
 & \qquad\qquad\qquad 18x - 36 \\
 & \qquad\qquad\qquad - \qquad + \\
\cline{2-2}
 & \qquad\qquad\qquad\qquad\quad 30
\end{array}
$$

∴ Quotient = $x^2 - \frac{11}{2}x + 18$, remainder = 30

Question 21 (3.0 marks)

Use the following information to answer the next question.

In the given figure, MN is the median of ΔAMC with respect to AC and area (ABMN) = area (ΔABL).	

Find the value of AM: NL with respect to the given figure.

Solution:

It is given that, area (ABMN) = area (ΔABL)

⇒ ar (ABMN) − ar (ΔABM) = ar (ΔABL) − ar (ΔABM)

⇒ ar (ΔAMN) = ar (ΔAML)

ΔAMN and ΔAML lie on the same base AM and they have equal areas.

Therefore, ΔAMN and ΔAML must lie between the same parallels.

i.e., AM||NL

It is given that MN is the median of ΔAMC with respect to AC.

i.e., N is the mid-point of AC.

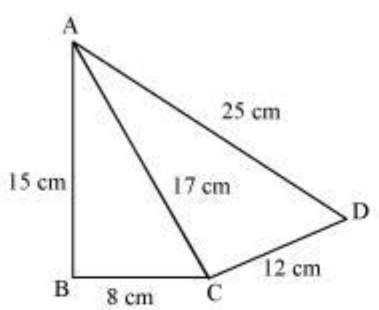
Therefore, by the converse of mid-point theorem, L is the mid-point of MC and $\text{NL} = \frac{\text{AM}}{2}$

$$\therefore \frac{\text{AM}}{\text{NL}} = \frac{2}{1}$$

Thus, the value of AM:NL is 2:1.

Question 22 (3.0 marks)

Find the area of the given quadrilateral ABCD.



Solution:
Area of quadrilateral ABCD = Area of ΔABC + Area of ΔACD

In ΔABC, it is observed that ABC is a right-angled triangle as it satisfies Pythagoras theorem.

$(AB)^2 + (BC)^2 = (15\text{ cm})^2 + (8\text{ cm})^2$

$= 225\text{ cm}^2 + 64\text{ cm}^2$

$= 289\text{ cm}^2$

$= (17\text{ cm})^2$

$= (AC)^2$

Therefore, by Pythagoras Theorem, ΔABC is right-angled at B.

$$\therefore \text{Area of } \Delta ABC = \frac{1}{2} \times \text{Base} \times \text{Height} = \frac{1}{2} \times BC \times AB = \frac{1}{2} \times 8\text{ cm} \times 15\text{ cm} = 60\text{ cm}^2$$

Consider ΔACD.

Here, $a = 12$ cm, $b = 25$ cm, $c = 17$ cm

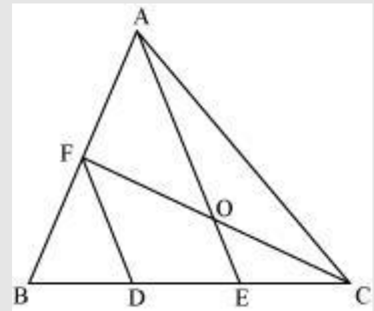
$$\therefore \text{Semi-perimeter } (s) = \frac{a+b+c}{2} = \frac{12+25+17}{2}\text{ cm} = 27\text{ cm}$$

Therefore, by using Heron's formula,

$$\begin{aligned}\text{Area of } \Delta ACD &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{27(27-12)(27-25)(27-17)} \\ &= \sqrt{27 \times 15 \times 2 \times 10} \\ &= \sqrt{3 \times 3 \times 3 \times 3 \times 5 \times 2 \times 2 \times 5} \\ &= 3 \times 3 \times 5 \times 2 \\ &= 90\text{ cm}^2\end{aligned}$$

Thus, area of quadrilateral ABCD = $60\text{ cm}^2 + 90\text{ cm}^2 = 150\text{ cm}^2$

Question 23 (3.0 marks)
Use the following information to answer the next question.

The given figure represents ΔABC. F is the mid-point of AB. From the vertex A, a line is drawn parallel to DF, intersecting BC at E.	

If AE = 12 cm and BE:EC = 2:1, then what is the length of OA?

Solution:
Consider ΔABE,

AF = FB [F is the mid-point of AB]

AE||DF

Therefore, by the converse of mid-point theorem, we obtain

D is the mid-point of BE. … (1)

$$\text{FD} = \frac{\text{AE}}{2} = \frac{12}{2}\text{ cm} = 6\text{ cm} \qquad \ldots(2)$$

It is given that BE:EC = 2:1

⇒ BE = 2EC

⇒ BD + DE = 2EC

⇒ 2DE = 2EC [Using (1)]

⇒ DE = EC

Therefore, in ΔCFD, it is observed that E is the mid-point of CD and FD||OE.

Therefore, again by the converse of mid-point theorem, O is the mid-point of FC and

$$\text{OE} = \frac{\text{DF}}{2}$$

$$\therefore \text{OE} = \frac{6}{2} \text{ cm} \qquad \left[\text{Using}(2)\right]$$
$$= 3 \text{ cm}$$

Thus, $\text{OA} = \text{AE} - \text{OE} = 12 \text{ cm} - 3 \text{ cm} = 9 \text{ cm}$

Question 24 (3.0 marks)

If $\frac{1}{\sqrt{7}+\sqrt{6}} + \frac{1}{2\sqrt{2}+\sqrt{7}} + \frac{1}{3+2\sqrt{2}} = a + b\sqrt{6}$, then find the value of $(a-2b)^2$.

Solution:

$$\frac{1}{\sqrt{7}+\sqrt{6}} = \frac{1}{\sqrt{7}+\sqrt{6}} \times \frac{\sqrt{7}-\sqrt{6}}{\sqrt{7}-\sqrt{6}} = \frac{\sqrt{7}-\sqrt{6}}{\left(\sqrt{7}+\sqrt{6}\right)\left(\sqrt{7}-\sqrt{6}\right)}$$
$$= \frac{\sqrt{7}-\sqrt{6}}{\left(\sqrt{7}\right)^2 - \left(\sqrt{6}\right)^2} = \frac{\sqrt{7}-\sqrt{6}}{7-6} = \sqrt{7}-\sqrt{6}$$

Similarly, $\frac{1}{2\sqrt{2}+\sqrt{7}} = 2\sqrt{2}-\sqrt{7}$ and $\frac{1}{3+2\sqrt{2}} = 3-2\sqrt{2}$

It is given that $\frac{1}{\sqrt{7}+\sqrt{6}} + \frac{1}{2\sqrt{2}+\sqrt{7}} + \frac{1}{3+2\sqrt{2}} = a + b\sqrt{6}$

$$\Rightarrow \left(\sqrt{7}-\sqrt{6}\right) + \left(2\sqrt{2}-\sqrt{7}\right) + \left(3-2\sqrt{2}\right) = a + b\sqrt{6}$$
$$\Rightarrow 3 - \sqrt{6} = a + b\sqrt{6}$$

Comparing both sides of the equation, we obtain $a = 3$, $b = -1$

Then, $(a-2b)^2 = \left[3-2(-1)\right]^2 = 5^2 = 25$

Question 25 (3.0 marks)

For three variables *a*, *b*, and *c*, if $a+b+c=10$, $a^2+b^2+c^2=38$, and $abc = 30$, then find the value of $a^3+b^3+c^3$.

Solution:

$$a+b+c=10$$
$$\Rightarrow (a+b+c)^2 = (10)^2$$
$$\Rightarrow a^2+b^2+c^2+2(ab+bc+ca) = 100$$
$$\Rightarrow 38 + 2(ab+bc+ca) = 100 \qquad \left[a^2+b^2+c^2=38\right]$$
$$\Rightarrow ab+bc+ca = \frac{100-38}{2} = 31$$

It is known that $a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

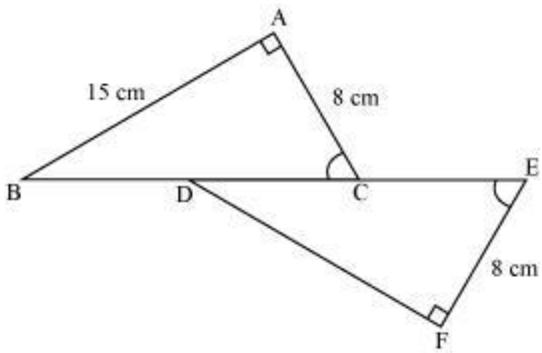
$\Rightarrow a^3+b^3+c^3-3\times 30=(a+b+c)\left[(a^2+b^2+c^2)-(ab+bc+ca)\right]$

$\Rightarrow a^3+b^3+c^3-90=10(38-31)=10\times 7=70$

$\Rightarrow a^3+b^3+c^3=70+90=160$

Question 26 (3.0 marks)

In the given figure, ∠ACB = ∠FED and BE = 25 cm. Find the length of DC?



Solution:

Applying Pythagoras Theorem in right-angled ΔABC, we obtain

$BC^2 = AB^2 + AC^2 = (15\text{ cm})^2 + (8\text{ cm})^2 = 225\text{ cm}^2 + 64\text{ cm}^2$

$\Rightarrow BC^2 = 289\text{ cm}^2 = (17\text{ cm})^2$

⇒ BC = 17 cm

In ΔABC and ΔFDE,

∠BAC = ∠DFE [Each is 90°]

∠ACB = ∠FED [Given]

AC = FE [Each is 8 cm]

∴ ΔABC ≅ ΔFDE [By ASA congruency criterion]

⇒ BC = DE [CPCT]

⇒ DE = 17 cm

It is given that BE = 25 cm

⇒ BC + CE = 25 cm

⇒ 17 cm + CE = 25 cm

⇒ CE = 25 cm − 17 cm = 8 cm

DE = 17 cm

⇒ DC + CE = 17 cm

⇒ DC + 8 cm = 17 cm

⇒ DC = 17 cm − 8 cm = 9 cm

Thus, the length of DC is 9 cm.

Question 27 (3.0 marks)
Simplify.

$$\sqrt[5]{\sqrt[4]{\left(2^4\right)^3}} - 3\sqrt[5]{8} + 4\sqrt[4]{\sqrt[5]{2^{12}}}$$.

Solution:
The given expression $\sqrt[5]{\sqrt[4]{\left(2^4\right)^3}} - 3\sqrt[5]{8} + 4\sqrt[4]{\sqrt[5]{2^{12}}}$ can be simplified as

$$\sqrt[5]{\sqrt[4]{\left(2^4\right)^3}} - 3\sqrt[5]{8} + 4\sqrt[4]{\sqrt[5]{2^{12}}}$$
$$= \sqrt[5]{\sqrt[4]{\left(2^3\right)^4}} - 3(8)^{\frac{1}{5}} + 4\sqrt[4]{\left(2^{12}\right)^{\frac{1}{5}}}$$
$$= \sqrt[5]{\left\{\left(2^3\right)^4\right\}^{\frac{1}{4}}} - 3\left(2^3\right)^{\frac{1}{5}} + 4\sqrt[4]{\left(2^{12}\right)^{\frac{1}{5}}}$$
$$= \sqrt[5]{\left(2^3\right)^{4\times\frac{1}{4}}} - 3(2)^{3\times\frac{1}{5}} + 4\sqrt[4]{2^{12\times\frac{1}{5}}} \qquad \left[\left(a^m\right)^n = a^{mn}\right]$$
$$= \sqrt[5]{2^3} - 3(2)^{\frac{3}{5}} + 4\sqrt[4]{2^{\frac{12}{5}}}$$
$$= (2)^{\frac{3}{5}} - 3(2)^{\frac{3}{5}} + 4\left(2^{\frac{12}{5}}\right)^{\frac{1}{4}}$$
$$= -2(2)^{\frac{3}{5}} + 4(2)^{\frac{12}{5}\times\frac{1}{4}}$$
$$= -2(2)^{\frac{3}{5}} + 4(2)^{\frac{3}{5}}$$
$$= 2(2)^{\frac{3}{5}} = (2)^{1+\frac{3}{5}} \qquad \left[a^m \times a^n = a^{m+n}\right]$$
$$= 2^{\frac{8}{5}}$$

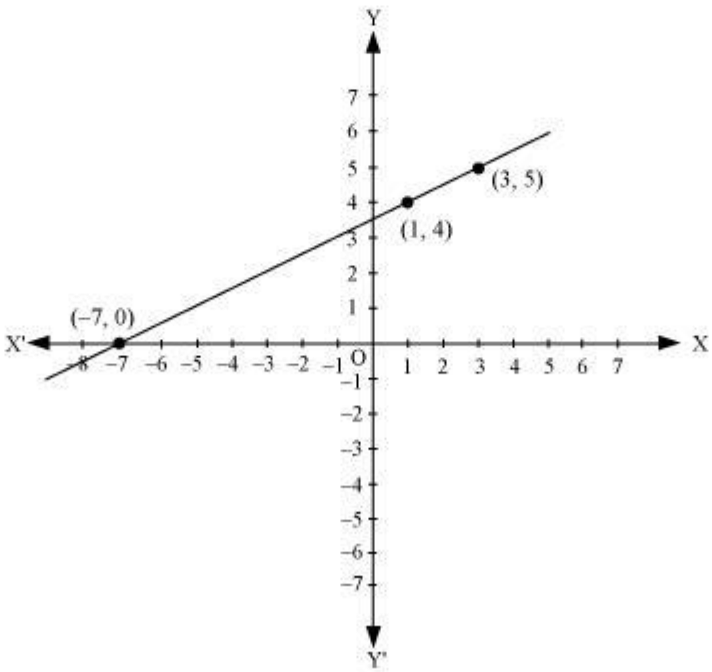
Question 28 (3.0 marks)
Determine whether the following sets of points form the vertices of a triangle.

(a) (−7, 0), (1, 4), and (3, 5)

(b) (−2, 4), (2, −4), and (6, 3)

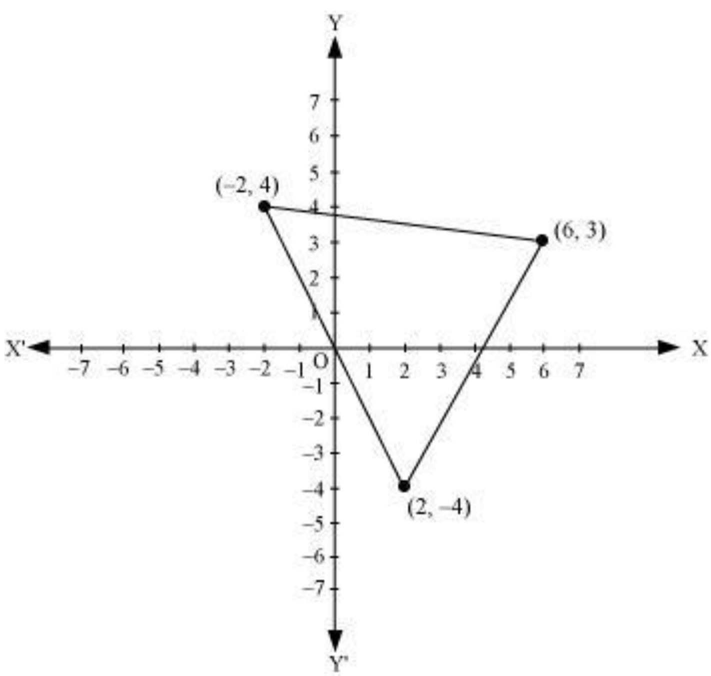
Solution:

(a) The points (−7, 0), (1, 4), and (3, 5) can be plotted on a graph as:



It is seen that, the given points lie on a line. So, they are not the vertices of triangle.

(b) The points (−2, 4), (2, −4), and (6, 3) can be plotted on a graph as:



Thus, the points (−2, 4), (2, −4), and (6, 3) are the vertices of a triangle.

Question 29 (3.0 marks)

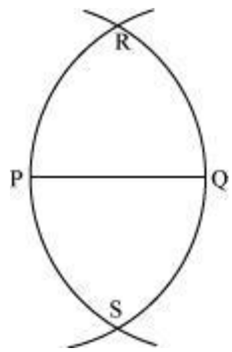
For a given line segment, use Euclidian Geometry to prove that a rhombus can be constructed having the line segment as its diagonal such that each side of the rhombus equals the length of the line segment.

Solution:
Consider a line segment, say PQ of any length.

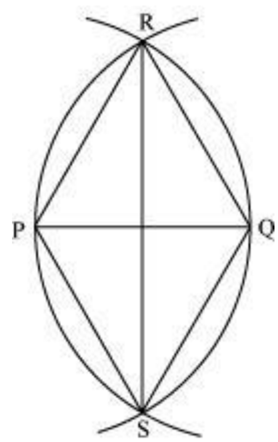
Euclid’s third postulate states that a circle can be drawn with any centre and any radius.

Using this postulate, draw a circle with point P as the centre and with PQ as the radius. Similarly, draw another circle with point Q as the centre and with QP as the radius.

Let the two circles intersect at points R and S.



Join P with R, P with S, Q with R, and Q with S.



Now, PR = PS = PQ (Radii of the same circle)

QR = QS = QP (Radii of the same circle)

One of Euclid’s axioms states that “things which are equal to the same thing are equal to one another”. Using this axiom:

PR = PS = QR = QS (= PQ)

Hence, PRQS is the required rhombus with diagonal as PQ and length of each side also equal to PQ.

Question 30 (4.0 marks)
Simplify.

$$\frac{(a-b)^2}{(b-c)(c-a)}+\frac{(b-c)^2}{(c-a)(a-b)}+\frac{(c-a)^2}{(a-b)(b-c)}$$

Solution:

The given expression $\frac{(a-b)^2}{(b-c)(c-a)}+\frac{(b-c)^2}{(c-a)(a-b)}+\frac{(c-a)^2}{(a-b)(b-c)}$ can be simplified as

Let $a-b=x,\ b-c=y,\ c-a=z$

Then, we have

$$\frac{(a-b)^2}{(b-c)(c-a)}+\frac{(b-c)^2}{(c-a)(a-b)}+\frac{(c-a)^2}{(a-b)(b-c)}$$
$$=\frac{x^2}{yz}+\frac{y^2}{zx}+\frac{z^2}{xy}$$
$$=\frac{x^3}{xyz}+\frac{y^3}{xyz}+\frac{z^3}{xyz}$$
$$=\frac{x^3+y^3+z^3}{xyz}$$

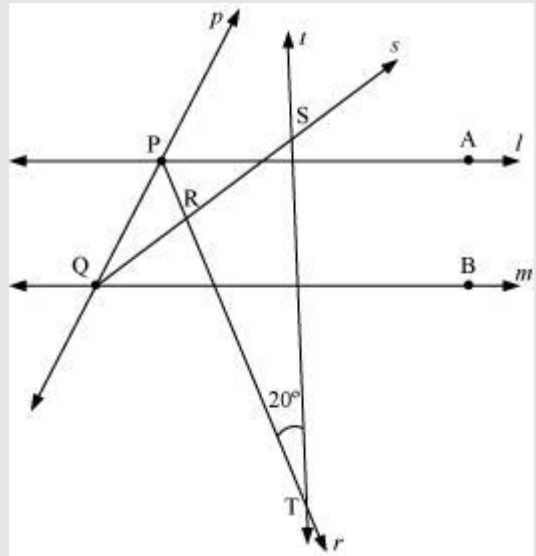
$$=\frac{\left[x^3+y^3+z^3-3xyz\right]+3xyz}{xyz}$$
$$=\frac{(x+y+z)\left(x^2+y^2+z^2-xy-yz-zx\right)+3xyz}{xyz}$$
$$=\frac{0\left(x^2+y^2+z^2-xy-yz-zx\right)+3xyz}{xyz} \qquad \left[x+y+z=(a-b)+(b-c)+(c-a)=0\right]$$

$$=\frac{3xyz}{xyz}$$
$$=3$$

Question 31 (4.0 marks)

Use the following information to answer the next question.

In the given figure, the lines *l* and *m* are parallel. Lines *r* and *s* are the bisectors of ∠APQ and ∠BQP respectively.



Find the measure of ∠QST?

Solution:
The lines *l* and *m* are parallel.

It is observed that ∠APQ and ∠BQP are interior angles on the same side of the transversal *p*.

∴ ∠APQ + ∠BQP = 180°

$$\Rightarrow \frac{1}{2}\angle APQ + \frac{1}{2}\angle BQP = 90°$$

$\Rightarrow \angle RPQ + \angle RQP = 90°$ [Lines *r* and *s* are the bisectors of ∠APQ and ∠BQP respectively]

Applying angle sum property of triangles in ΔPQR, we obtain

(∠RPQ + ∠RQP) + ∠PRQ = 180°

⇒ 90° + ∠PRQ = 180°

⇒ ∠PRQ = 180° − 90° = 90°

∠PRQ and ∠SRT are vertically opposite angles.

∴ ∠SRT = ∠PRQ = 90°

Again applying angle sum property of triangles in ΔRST, we obtain

∠SRT + ∠RTS + ∠TSR = 180°

$\Rightarrow 90° + 20° + \angle QST = 180°$

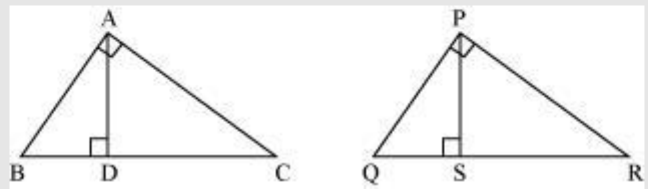
$\Rightarrow 110° + \angle QST = 180°$

$\Rightarrow \angle QST = 180° - 110° = 70°$

Thus, the measure of $\angle QST$ is 70°.

Question 32 (4.0 marks)
Use the following information to answer the next question.

The given figure shows two triangles ABC and PQR, where $\angle BAC = \angle QPR = 90°$. D and S are points on the side BC and QR respectively such that $AD \perp BC$ and $PS \perp QR$.



If AB = PQ = 6 cm, AC = 8 cm, AD = 4.8 cm, and QS = 3.6 cm, then what is the perimeter of ΔPQR?

Solution:
ΔABC is right-angled at A.

Applying Pythagoras Theorem in ΔABC, we obtain

$\therefore BC^2 = AB^2 + AC^2 = (6 \text{ cm})^2 + (8 \text{ cm})^2 = 100 \text{ cm}^2$

$\Rightarrow BC = 10$ cm

ΔABD is right-angled at D.

Applying Pythagoras Theorem in ΔABD, we obtain

$\therefore BD^2 = AB^2 - AD^2 = (6 \text{ cm})^2 - (4.8 \text{ cm})^2 = 12.96 \text{ cm}^2$

$\Rightarrow BD = 3.6$ cm

Comparing ΔABD and ΔPQS, we obtain

AB = PQ [Each is 6 cm]

$\angle ADB = \angle PSQ$ [Each is 90°]

BD = QS [Each is 3.6 cm]

$\therefore \Delta ABD \cong \Delta PQS$ [By RHS congruency criterion]

$\Rightarrow \angle ABD = \angle PQS$ [CPCT]

Now, comparing ΔABC and ΔPQR, we obtain

$\angle BAC = \angle QPR$ [Each is 90°]

AB = PQ [Each is 6 cm]

∠ABC = ∠PQR [∠ABD = ∠PQS]

∴ ΔABC ≅ ΔPQR [By ASA congruency criterion]

⇒ AC = PR = 8 cm and BC = QR = 10 cm [CPCT]

Thus, perimeter of ΔPQR = PQ + QR + PR = 6 cm + 10 cm + 8 cm = 24 cm

Question 33 (4.0 marks)
Factorize the polynomial $p(x) = x^3 + 13x^2 + 32x + 20$

Solution:
$p(x) = x^3 + 13x^2 + 32x + 20$

By hit and trial method, substituting $x = -1$ in $p(x)$, we obtain

$p(-1) = (-1)^3 + 13(-1)^2 + 32(-1) + 20 = 0$

Therefore, by factor theorem, we obtain $(x + 1)$ as a factor of $p(x)$.

Dividing $p(x)$ by $(x + 1)$,

$$
\begin{array}{r|l}
 & x^2 + 12x + 20 \\
\hline
x+1 & x^3 + 13x^2 + 32x + 20 \\
 & x^3 + x^2 \\
 & - \quad - \\
\hline
 & 12x^2 + 32x + 20 \\
 & 12x^2 + 12x \\
 & - \quad\quad - \\
\hline
 & 20x + 20 \\
 & 20x + 20 \\
\hline
 & 0 \\
\hline
\end{array}
$$

$$\therefore p(x) = (x+1)(x^2 + 12x + 20)$$

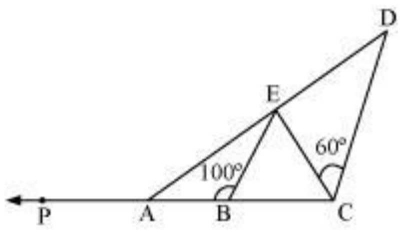
Splitting the middle term,

$p(x) = (x + 1)(x^2 + 10x + 2x + 20) = (x + 1)(x + 10)(x + 2)$

Question 34 (4.0 marks)
Prove that the sum of the angles of a triangle is 180°. Deduce that if a side of a triangle is produced, then the exterior angle so formed is equal to the sum of the two interior opposite angles.

Using this, answer the following question:

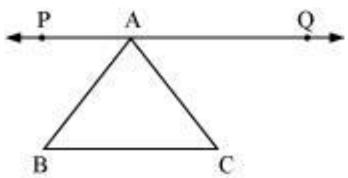
In the given figure, BE||CD and CE⊥AD. What is the measure of ∠PAD?



Solution:
Given: ΔABC

To prove: $\angle BAC + \angle ABC + \angle BCA = 180°$

Construction: Draw a line PAQ parallel to BC passing through A.



Proof: $PQ \parallel BC$

$\therefore \angle PAB = \angle B$ … (1) (Alternate interior angles)

$\angle QAC = \angle C$ … (2) (Alternate interior angles)

Now, $\angle PAB + \angle BAQ = 180°$ (Linear pair)

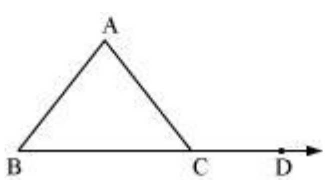
$\therefore \angle PAB + (\angle BAC + \angle QAC) = 180°$

$\Rightarrow \angle B + \angle BAC + \angle C = 180°$ [Using (1) and (2)]

$\therefore \angle BAC + \angle ABC + \angle BCA = 180°$

Thus, the sum of the angles of a triangle is 180°.

Now, extend BC to point D.



$\angle A + \angle B + \angle BCA = 180°$ … (3) (Using above result)

Also, $\angle BCA + \angle ACD = 180°$ … (4) (Linear pair)

From (3) and (4):

$\angle A + \angle B + \angle BCA = \angle BCA + \angle ACD$

$\therefore \angle ACD = \angle A + \angle B$

Thus, if a side of a triangle is produced, then the exterior angle so formed is equal to the sum of the two interior opposite angles.

In the given figure, $BE \parallel CD$

$\therefore \angle BEC = \angle ECD = 60°$ … (A) (Alternate interior angles)

$\angle AEC = 90°$ (Given)

$\Rightarrow \angle BEC + \angle AEB = 90°$

$\Rightarrow 60° + \angle AEB = 90°$ [Using (A)]

$\Rightarrow \angle AEB = 30°$ … (B)

In ΔABE, $\angle PAE = \angle AEB + \angle ABE$ (Using the proved result)

$\therefore \angle PAE = 30° + 100° = 130°$

$\therefore \angle PAD = 130°$