Number Systems (Math)

Exercise 1.3

Question 1:

Write the following in decimal form and say what kind of decimal expansion each has:

(i) $\frac{36}{100}$ (ii) $\frac{1}{11}$ (iii) $4\frac{1}{8}$

(iv) $\frac{3}{13}$ (v) $\frac{2}{11}$ (vi) $\frac{329}{400}$

Answer:

(i) $\frac{36}{100} = 0.36$

Terminating

(ii) $\frac{1}{11} = 0.090909..... = 0.\overline{09}$

Non-terminating repeating

(iii) $4\frac{1}{8} = \frac{33}{8} = 4.125$

Terminating

(iv) $\frac{3}{13} = 0.230769230769.... = 0.\overline{230769}$

Non-terminating repeating

(v) $\frac{2}{11} = 0.18181818..... = 0.\overline{18}$

Non-terminating repeating

(vi) $\frac{329}{400} = 0.8225$

Terminating

Question 2:

You know that. $\frac{1}{7} = 0.\overline{142857}$ Can you predict what the decimal expansion of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$ are, without actually doing the long division? If so, how?

[**Hint:** Study the remainders while finding the value of $\frac{1}{7}$ carefully.]

Answer:

Yes. It can be done as follows.

$$\frac{2}{7} = 2 \times \frac{1}{7} = 2 \times 0.\overline{142857} = 0.\overline{285714}$$

$$\frac{3}{7} = 3 \times \frac{1}{7} = 3 \times 0.\overline{142857} = 0.\overline{428571}$$

$$\frac{4}{7} = 4 \times \frac{1}{7} = 4 \times 0.\overline{142857} = 0.\overline{571428}$$

$$\frac{5}{7} = 5 \times \frac{1}{7} = 5 \times 0.\overline{142857} = 0.\overline{714285}$$

$$\frac{6}{7} = 6 \times \frac{1}{7} = 6 \times 0.\overline{142857} = 0.\overline{857142}$$

Question 3:

Express the following in the form, $\frac{p}{q}$ where p and q are integers and $q \neq 0$.

(i) $0.\overline{6}$ (ii) $0.4\overline{7}$ (iii) $0.\overline{001}$

Answer:

(i) $0.\overline{6} = 0.666...$

Let $x = 0.666\ldots$

$10x = 6.666\ldots$

$10x = 6 + x$

$9x = 6$

$$x = \frac{2}{3}$$

(ii) $0.\overline{47} = 0.4777.....$

$$= \frac{4}{10} + \frac{0.777}{10}$$

Let $x = 0.777\ldots$

$10x = 7.777\ldots$

$10x = 7 + x$

$$x = \frac{7}{9}$$

$$\frac{4}{10} + \frac{0.777...}{10} = \frac{4}{10} + \frac{7}{90}$$

$$= \frac{36+7}{90} = \frac{43}{90}$$

(iii) $0.\overline{001} = 0.001001...$

Let $x = 0.001001\ldots$

$1000x = 1.001001\ldots$

$1000x = 1 + x$

$999x = 1$

$$x = \frac{1}{999}$$

Question 4:

Express 0.99999…in the form. Are you surprised by your answer? With your teacher and classmates discuss why the answer makes s$\frac{p}{q}$e.

Answer:

Let $x = 0.9999\ldots$

$10x = 9.9999\ldots$

$10x = 9 + x$

$9x = 9$

$x = 1$

Question 5:

What can the maximum number of digits be in the repeating block of digits in the decimal expansion of $\frac{1}{17}$? Perform the division to check your answer.

Answer:

It can be observed that,

$$\frac{1}{17} = 0.\overline{0588235294117647}$$

There are 16 digits in the repeating block of the decimal expansion of. $\frac{1}{17}$

Question 6:

Look at several examples of rational numbers in the form $\frac{p}{q}$ ($q \neq 0$), where p and q are integers with no common factors other than 1 and having terminating decimal representations (expansions). Can you guess what property q must satisfy?

Answer:

Terminating decimal expansion will occur when denominator q of rational number $\frac{p}{q}$ is either of 2, 4, 5, 8, 10, and so on…

$$\frac{9}{4} = 2.25$$
$$\frac{11}{8} = 1.375$$
$$\frac{27}{5} = 5.4$$

It can be observed that terminating decimal may be obtained in the situation where prime factorization of the denominator of the given fractions has the power of 2 only or 5 only or both.

Question 7:

Write three numbers whose decimal expansions are non-terminating non-recurring.

Answer:

3 numbers whose decimal expansions are non-terminating non-recurring are as follows.

0.505005000500005000005…

0.72072007200072000072000000…

0.0800800080000800000800000008…

Question 8:

Find three different irrational numbers between the rational numbers and.

$$\frac{5}{7} \qquad \frac{9}{11}$$

Answer:

$$\frac{5}{7} = 0.\overline{714285}$$

$$\frac{9}{11} = 0.\overline{81}$$

3 irrational numbers are as follows.

0.73073007300073000073…

0.75075007500075000075…

0.79079007900079000079…

Question 9:

Classify the following numbers as rational or irrational:

(i) $\sqrt{23}$ (ii) $\sqrt{225}$ (iii) 0.3796

(iv) 7.478478 (v) 1.101001000100001…

Answer:

(i) $\sqrt{23} = 4.79583152331 \ldots$

As the decimal expansion of this number is non-terminating non-recurring, therefore, it is an irrational number.

(ii) $\sqrt{225} = 15 = \frac{15}{1}$

It is a rational number as it can be represented in $\frac{p}{q}$ form.

(iii) 0.3796

As the decimal expansion of this number is terminating, therefore, it is a rational number.

(iv) 7.478478 … $= 7.\overline{478}$

As the decimal expansion of this number is non-terminating recurring, therefore, it is a rational number.

(v) 1.10100100010000 …

As the decimal expansion of this number is non-terminating non-repeating, therefore, it is an irrational number.