Number Systems Question Answers Test-1

Subjective Test

Question 1 (1.0 marks)

State whether the following statements are true or false.

(a) The sum of a rational number and an irrational number is a rational number. ($\frac{1}{2}$ mark)

(b) The product of a rational number and an irrational number is an irrational number. (1 mark)

Solution:

(a) False. The sum of a rational number and an irrational number is always an irrational number.

(b) True

Question 2 (2.0 marks)

Without actual division, state whether each of the following fractions is a terminating decimal or not. Give reasons to justify your answer.

(a) $\frac{3}{40}$ (1 mark)

(b) $\frac{13}{42}$ (1 mark)

Solution:

(a) For the fraction, $\frac{3}{40}$ the numerator (3) and the denominator (40) are co-prime.

Now, the denominator of $\frac{3}{40}$ is 40, which can be prime factorized as $40 = 2^3 \times 5$.

As seen in the prime factorization of 40, it has no prime factors other than 2 and 5.

Therefore, the fraction $\frac{3}{40}$ is a terminating decimal.

(b) For the fraction, $\frac{13}{42}$ the numerator (13) and the denominator (42) are co-prime.

Now, the denominator of $\frac{13}{42}$ is 42, which can be prime factorised as $42 = 2 \times 3 \times 7$.

As seen in the prime factorization of 42, it has prime factors (3 and 7) other than 2 and 5 as well.

Therefore, the fraction $\frac{13}{42}$ is a non-terminating decimal.

Question 3 (2.0 marks)

Express $0.43\overline{7}$ as a fraction in the simplest form.

Solution:

Let $y = 0.43\overline{7}$

Then, $y = 0.43777777\ldots$

On multiplying both sides by 100, we get

$100y = 43.777777777\ldots$ (1)

On multiplying both sides of equation (1) by 10, we get

$1000y = 437.777777\ldots$ (2)

On subtracting equation (1) from equation (2), we get

$900y = 394$

$$y = \frac{394}{900}$$

$$y = \frac{197}{450}$$

Thus, $0.43\overline{7}$ can be expressed as a fraction in the simplest form as $\frac{197}{450}$

Question 4 (2.0 marks)

Find eight rational numbers between 7 and 8.

Solution:

Eight rational numbers are required to be calculated.

Thus, the two given numbers (7 and 8) can be rewritten with denominator as 8 + 1 = 9 as $7 = \frac{63}{9}$ and $8 = \frac{72}{9}$

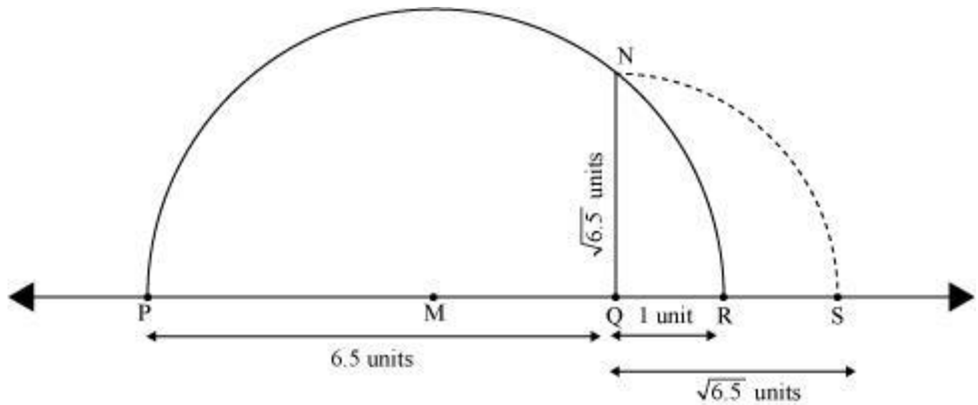
Now, eight rational numbers between $\frac{63}{9}$ and $\frac{72}{9}$ are $\frac{64}{9}$, $\frac{65}{9}$, $\frac{66}{9}$, $\frac{67}{9}$, $\frac{68}{9}$, $\frac{69}{9}$, $\frac{70}{9}$ and $\frac{71}{9}$

Thus, eight rational numbers between 7 and 8 are. $\frac{64}{9}$, $\frac{65}{9}$, $\frac{66}{9}$, $\frac{67}{9}$, $\frac{68}{9}$, $\frac{69}{9}$, $\frac{70}{9}$ and $\frac{71}{9}$

Question 5 (3.0 marks)

Represent $\sqrt{6.5}$ on the number line.

Solution:



Draw a line segment PQ = 6.5 units and extend it to R such that QR = 1 unit.

M is the midpoint of PR.

With M as the centre and MR as the radius, draw a semicircle.

Draw NQ ⊥ PR, intersecting the semicircle at N.

Then, QN = $\sqrt{6.5}$ units

Now, with Q as the centre and QN as the radius, draw an arc, meeting PR at S (when PR is extended).

Thus, QS = QN = $\sqrt{6.5}$ units

Question 6 (3.0 marks)

If, $x = \sqrt{3} - 1$ then find the value of. $\left\{x + \frac{1}{x}\right\}^2$

Solution: $x = \sqrt{3} - 1$

$$\Rightarrow \frac{1}{x} = \frac{1}{\sqrt{3}-1}$$

$$\Rightarrow \frac{1}{x} = \frac{1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$\Rightarrow \frac{1}{x} = \frac{\sqrt{3}+1}{\left(\sqrt{3}\right)^2 - (1)^2}$$

$$\Rightarrow \frac{1}{x} = \frac{\sqrt{3}+1}{3-1}$$

$$\Rightarrow \frac{1}{x} = \frac{\sqrt{3}+1}{2}$$

$$\Rightarrow \left(x+\frac{1}{x}\right)^2 = \left(\sqrt{3}-1+\frac{\sqrt{3}+1}{2}\right)^2$$

$$\Rightarrow \left(x+\frac{1}{x}\right)^2 = \left(\frac{2\sqrt{3}-2+\sqrt{3}+1}{2}\right)^2$$

$$\Rightarrow \left(x+\frac{1}{x}\right)^2 = \left(\frac{3\sqrt{3}-1}{2}\right)^2$$

$$\Rightarrow \left(x+\frac{1}{x}\right)^2 = \left(\frac{\left(3\sqrt{3}\right)^2+(1)^2-2\times 3\sqrt{3}\times 1}{(2)^2}\right)$$

$$\Rightarrow \left(x+\frac{1}{x}\right)^2 = \frac{27+1-6\sqrt{3}}{4} = \frac{28-6\sqrt{3}}{4}$$

$$\Rightarrow \left(x+\frac{1}{x}\right)^2 = \frac{2\left(14-3\sqrt{3}\right)}{4} = \frac{14-3\sqrt{3}}{2}$$

Thus, the value of $\left(x+\frac{1}{x}\right)^2$ is $\frac{14-3\sqrt{3}}{2}$.

Question 7 (6.0 marks)

Simplify the following expressions.

(a) $\dfrac{(25)^{\frac{1}{2}}\times(32)^{\frac{2}{5}}}{(27)^{\frac{2}{3}}\times(16)^{\frac{1}{4}}}$ (2 marks)

(b) $\left(\dfrac{27}{8}\right)^{-\frac{1}{3}}\times\left[\left(\dfrac{125}{27}\right)^{-\frac{4}{3}}\div\left(\dfrac{2}{5}\right)^4\right]$ (2 marks)

(c) $\dfrac{(81)^{-\frac{7}{4}}-(81)^{-\frac{3}{4}}}{(81)^{\frac{5}{4}}}$ (2 marks)

Solution:

(a)

$$\frac{(25)^{\frac{1}{2}}\times(32)^{\frac{2}{5}}}{(27)^{\frac{2}{3}}\times(16)^{\frac{1}{4}}} = \frac{\left(5^2\right)^{\frac{1}{2}}\times\left(2^5\right)^{\frac{2}{5}}}{\left(3^3\right)^{\frac{2}{3}}\times\left(2^4\right)^{\frac{1}{4}}}$$

$$= \frac{5^{2\times\frac{1}{2}} \times 2^{5\times\frac{2}{5}}}{3^{3\times\frac{2}{3}} \times 2^{4\times\frac{1}{4}}} \qquad \left[\left(a^x\right)^y = a^{x\times y}\right]$$

$$= \frac{5^1 \times 2^2}{3^2 \times 2^1}$$

$$= \frac{5\times 2\times 2}{3\times 3\times 2}$$

$$= \frac{20}{18}$$

$$= \frac{10}{9}$$

(b) $\left(\frac{27}{8}\right)^{-\frac{1}{3}} \times \left[\left(\frac{125}{27}\right)^{-\frac{4}{3}} \div \left(\frac{2}{5}\right)^4\right] = \left[\left(\frac{3}{2}\right)^3\right]^{-\frac{1}{3}} \times \left[\left\{\left(\frac{5}{3}\right)^3\right\}^{-\frac{4}{3}} \div \left(\frac{2}{5}\right)^4\right]$

$$= \left[\left(\frac{3}{2}\right)^{3\times\left(-\frac{1}{3}\right)}\right] \times \left[\left\{\left(\frac{5}{3}\right)^{3\times\left(-\frac{4}{3}\right)}\right\} \div \left(\frac{2}{5}\right)^4\right] \qquad \left[\left(a^x\right)^y = (a)^{x\times y}\right]$$

$$= \left[\left(\frac{3}{2}\right)^{-1}\right] \times \left[\left\{\left(\frac{5}{3}\right)^{-4}\right\} \div \left(\frac{2}{5}\right)^4\right]$$

$$= \frac{2}{3} \times \left[\left\{\left(\frac{5}{3}\right)^{-4}\right\} \div \left(\frac{2}{5}\right)^4\right] \qquad \left[\left(\frac{a}{b}\right)^{-1} = \frac{b}{a}\right]$$

$$= \frac{2}{3} \times \left[\left\{\left(\frac{5}{3}\right)^{-1}\right\}^4 \div \left(\frac{2}{5}\right)^4\right] \qquad \left[a^{x\times y} = \left(a^x\right)^y\right]$$

$$= \frac{2}{3} \times \left[\left(\frac{3}{5}\right)^4 \div \left(\frac{2}{5}\right)^4\right] \qquad \left[\left(\frac{a}{b}\right)^{-1} = \frac{b}{a}\right]$$

$$= \frac{2}{3} \times \left(\frac{3}{5} \div \frac{2}{5}\right)^4 \qquad \left[\frac{x^a}{y^a} = \left(\frac{x}{y}\right)^a\right]$$

$$= \frac{2}{3} \times \left(\frac{3}{5} \times \frac{5}{2}\right)^4$$

$$= \frac{2}{3} \times \left(\frac{3}{2}\right)^4$$

$$= \frac{2}{3} \times \frac{3\times 3\times 3\times 3}{2\times 2\times 2\times 2}$$

$$= \frac{3\times 3\times 3}{2\times 2\times 2}$$

$$= \frac{27}{8}$$

(c) $$\frac{(81)^{-\frac{7}{4}}-(81)^{-\frac{3}{4}}}{(81)^{\frac{5}{4}}}=\frac{\left(3^4\right)^{-\frac{7}{4}}-\left(3^4\right)^{-\frac{3}{4}}}{\left(3^4\right)^{\frac{5}{4}}}$$

$$=\frac{3^{4\times\left(-\frac{7}{4}\right)}-3^{4\times\left(-\frac{3}{4}\right)}}{3^{4\times\frac{5}{4}}} \qquad \left[\left(a^x\right)^y=a^{x\times y}\right]$$

$$=\frac{3^{-7}-3^{-3}}{3^5}$$

$$=\frac{3^{-7}}{3^5}-\frac{3^{-3}}{3^5}$$

$$=\frac{1}{3^{5+7}}-\frac{1}{3^{5+3}} \qquad \left[\frac{x^a}{x^b}=x^{a-b}=\frac{1}{x^{b-a}}\right]$$

$$=\frac{1}{3^{12}}-\frac{1}{3^8}$$

$$=\frac{1}{3^8}\left(\frac{1}{3^4}-1\right)$$

$$=\frac{1}{3^8}\left(\frac{1}{81}-1\right)$$

$$=\frac{1}{3^8}\left(\frac{-80}{81}\right)$$

$$=\frac{-80}{3^8\times3^4}$$

$$=\frac{-80}{3^{8+4}} \qquad \left[x^a\times x^b=x^{a+b}\right]$$

$$=\frac{-80}{3^{12}}$$

Question 8 (6.0 marks)

Simplify the following expressions.

(a) $\left(\frac{x^r}{x^p}\right)^q\left(\frac{x^q}{x^r}\right)^p\left(\frac{x^p}{x^q}\right)^r$ (2 marks)

(b) $\left(\frac{16}{9}\right)^{-\frac{1}{2}}\div\left[\left(\frac{256}{81}\right)^{-\frac{1}{4}}+\frac{\sqrt{3}}{\sqrt{27}}\right]$ (2 marks)

(c) $\left(\sqrt[3]{3}\right)^{-\frac{3}{2}}\times\sqrt[4]{b^4}\div\sqrt{a^2b}$ (2 marks)

Solution:

(a) $\left(\dfrac{x^r}{x^p}\right)^q \left(\dfrac{x^q}{x^r}\right)^p \left(\dfrac{x^p}{x^q}\right)^r = \left(x^{r-p}\right)^q \left(x^{q-r}\right)^p \left(x^{p-q}\right)^r$ $\qquad \left[\dfrac{x^a}{x^b} = x^{a-b}\right]$

$= x^{(r-p)q} \times x^{(q-r)p} \times x^{(p-q)r}$ $\qquad \left[\left(x^a\right)^b = x^{ab}\right]$

$= x^{qr-pq} \times x^{pq-rp} \times x^{rp-qr}$

$= x^{qr-pq+pq-rp+rp-qr}$ $\qquad \left[x^a \times x^b = x^{a+b}\right]$

$= x^0$

$= 1$ $\qquad \left[x^0 = 1\right]$

(b) $\left(\dfrac{16}{9}\right)^{-\frac{1}{2}} \div \left[\left(\dfrac{256}{81}\right)^{-\frac{1}{4}} + \dfrac{\sqrt{3}}{\sqrt{27}}\right] = \left(\dfrac{4^2}{3^2}\right)^{-\frac{1}{2}} \div \left[\left(\dfrac{4^4}{3^4}\right)^{-\frac{1}{4}} + \sqrt{\dfrac{3}{27}}\right]$ $\qquad \left[\dfrac{\sqrt{a}}{\sqrt{b}} = \sqrt{\dfrac{a}{b}}\right]$

$= \left[\left(\dfrac{4}{3}\right)^2\right]^{-\frac{1}{2}} \div \left[\left\{\left(\dfrac{4}{3}\right)^4\right\}^{-\frac{1}{4}} + \sqrt{\dfrac{1}{9}}\right]$

$= \left(\dfrac{4}{3}\right)^{2\times\left(-\frac{1}{2}\right)} \div \left[\left(\dfrac{4}{3}\right)^{4\times\left(-\frac{1}{4}\right)} + \dfrac{1}{3}\right]$ $\qquad \left[\left(a^x\right)^y = a^{xy}\right]$

$= \left(\dfrac{4}{3}\right)^{-1} \div \left[\left(\dfrac{4}{3}\right)^{-1} + \dfrac{1}{3}\right]$

$= \dfrac{3}{4} \div \left(\dfrac{3}{4} + \dfrac{1}{3}\right)$ $\qquad \left[a^{-1} = \dfrac{1}{a}\right]$

$= \dfrac{3}{4} \div \dfrac{13}{12}$

$= \dfrac{3}{4} \times \dfrac{12}{13}$

$= \dfrac{9}{13}$

(c) $\left(\sqrt[3]{3}\right)^{-\frac{3}{2}} \times \sqrt[4]{b^4} \div \sqrt{a^2 b} = \left(3^{\frac{1}{3}}\right)^{-\frac{3}{2}} \times \left(b^4\right)^{\frac{1}{4}} \div a\sqrt{b}$

$$= 3^{\frac{1}{3}\times\left(-\frac{3}{2}\right)} \times b^{4\times\frac{1}{4}} \div ab^{\frac{1}{2}} \qquad \left[\left(a^x\right)^y = a^{xy}\right]$$

$$= 3^{-\frac{1}{2}} \times b^1 \div ab^{\frac{1}{2}}$$

$$= \frac{b}{3^{\frac{1}{2}}} \times \frac{1}{ab^{\frac{1}{2}}} \qquad \left[a^{-x} = \frac{1}{a^x}\right]$$

$$= \frac{b^{1-\frac{1}{2}}}{3^{\frac{1}{2}}} \times \frac{1}{a} \qquad \left[\frac{a^x}{a^y} = a^{x-y}\right]$$

$$= \frac{b^{\frac{1}{2}}}{3^{\frac{1}{2}}} \times \frac{1}{a}$$

$$= \left(\frac{b}{3}\right)^{\frac{1}{2}} \times \frac{1}{a}$$

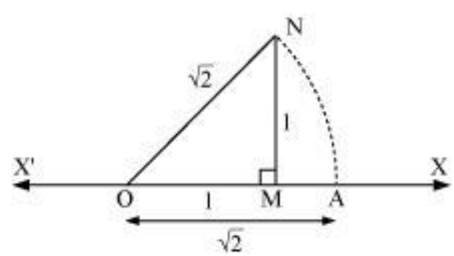
$$= \frac{1}{a}\sqrt{\frac{b}{3}}$$

Question 9 (6.0 marks)

Represent both $\sqrt{2}$ and $\sqrt{7}$ on the number line.

Solution:

Let XOX' be the horizontal axis (*x*-axis) and let O be the origin.



Take OM = 1 unit (M being any point on *x*-axis), and draw MN ⊥ OM such that MN = 1 unit.

Join ON.

In right triangle OMN:

$ON^2 = OM^2 + MN^2$

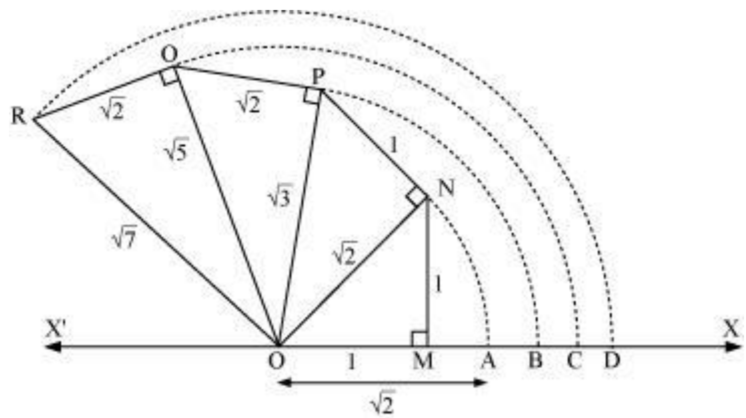
ON = $\sqrt{1^2 + 1^2}$ = $\sqrt{2}$ units

With O as the centre and ON as the radius, draw an arc, meeting XOX’ at A.

Then, OA = ON = $\sqrt{2}$ units

Thus, the point A represents $\sqrt{2}$ on the real number line.

Now, $\sqrt{7}$ can be represented on the number line in the same way.



Draw NP ⊥ ON such that NP = 1 unit.

Join OP

In the right triangle ONP:

OP = $\sqrt{ON^2 + NP^2}$

$OP = \sqrt{3}$ units

With O as the centre and OP as the radius, draw an arc, meeting XOX' at B.

Thus, OB = OP = $\sqrt{3}$ units

Draw PQ ⊥ OP such that PQ = $\sqrt{2}$ units.

Join OQ

In the right triangle OPQ:

OQ = $\sqrt{OP^2 + PQ^2} = \sqrt{3+2} = \sqrt{5}$ units

With O as the centre and OQ as the radius, draw an arc, meeting XOX' at C.

Thus, OC = OQ = $\sqrt{5}$ units

Now, draw QR ⊥ OQ such that QR = $\sqrt{2}$ units.

Join OR

In the right triangle OQR

OR = $\sqrt{OQ^2 + QR^2}$ = $\sqrt{7}$ units

With O as the centre and OR as the radius, draw an arc, meeting XOX' at D.

Thus, OD = OR = $\sqrt{7}$ units

Thus, point D represents $\sqrt{7}$ on the real number line.

Question 10 (6.0 marks)

Rationalize the denominators of the following expressions.

(a) $\dfrac{1}{\sqrt{5}+\sqrt{13}}$ (2 marks)

(b) $\dfrac{3-\sqrt{2}}{\sqrt{7}}$ (2 marks)

(c) $\dfrac{3\sqrt{6}-2\sqrt{5}}{3\sqrt{5}-6\sqrt{3}}$ (2 marks)

Solution:

(a) On multiplying the numerator and the denominator of $\dfrac{1}{\sqrt{5}+\sqrt{13}}$ by $\left(\sqrt{5}-\sqrt{13}\right)$, we get:

$$\frac{1}{\sqrt{5}+\sqrt{13}} = \frac{1}{\sqrt{5}+\sqrt{13}} \times \frac{\sqrt{5}-\sqrt{13}}{\sqrt{5}-\sqrt{13}}$$

$$= \frac{\sqrt{5}-\sqrt{13}}{5-13}$$

$$= \frac{\sqrt{5}-\sqrt{13}}{-8}$$

$$= \frac{1}{8}\left(\sqrt{13}-\sqrt{5}\right)$$

(b) On multiplying the numerator and the denominator of $\dfrac{3-\sqrt{2}}{\sqrt{7}}$ by $\sqrt{7}$, we get:

$$\frac{3-\sqrt{2}}{\sqrt{7}} = \frac{\left(3-\sqrt{2}\right)}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{3\sqrt{7}-\sqrt{14}}{7}$$

(c) On multiplying the numerator and the denominator of $\dfrac{3\sqrt{6}-2\sqrt{5}}{3\sqrt{5}-6\sqrt{3}}$ by, $\left(3\sqrt{5}+6\sqrt{3}\right)$ we get:

$$\frac{3\sqrt{6}-2\sqrt{5}}{3\sqrt{5}-6\sqrt{3}} = \frac{3\sqrt{6}-2\sqrt{5}}{3\sqrt{5}-6\sqrt{3}} \times \frac{3\sqrt{5}+6\sqrt{3}}{3\sqrt{5}+6\sqrt{3}}$$

$$= \frac{9\sqrt{30}+18\sqrt{18}-30-12\sqrt{15}}{45-108}$$

$$= \frac{9\sqrt{30}-30+54\sqrt{2}-12\sqrt{15}}{-63}$$

$$= \frac{3\left(10+4\sqrt{15}-18\sqrt{2}-3\sqrt{30}\right)}{3\times 21}$$

$$= \frac{1}{21}\left(10+4\sqrt{15}-18\sqrt{2}-3\sqrt{30}\right)$$