Number Systems (Math)

Exercise 1.5 Page 24

Question 1:

1Classify the following numbers as rational or irrational:

(i) $2-\sqrt{5}$ (ii)$\left(3+\sqrt{23}\right)-\sqrt{23}$ (iii) $\frac{2\sqrt{7}}{7\sqrt{7}}$

(iv) $\frac{1}{\sqrt{2}}$ (v) 2π

Answer:

(i) $2-\sqrt{5}$ $= 2 - 2.2360679\ldots$

$= -0.2360679\ldots$

As the decimal expansion of this expression is non-terminating non-recurring, therefore, it is an irrational number.

(ii) $\left(3+\sqrt{23}\right)-\sqrt{23}=3=\frac{3}{1}$

As it can be represented in $\frac{p}{q}$ form, therefore, it is a rational number.

(iii) $\frac{2\sqrt{7}}{7\sqrt{7}}=\frac{2}{7}$

As it can be represented in $\frac{p}{q}$ form, therefore, it is a rational number.

(iv) $\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}=0.7071067811\ldots$

As the decimal expansion of this expression is non-terminating non-recurring, therefore, it is an irrational number.

(v) $2\pi = 2(3.1415\ \ldots)$

$= 6.2830\ \ldots$

As the decimal expansion of this expression is non-terminating non-recurring, therefore, it is an irrational number.

Question 2:

Simplify each of the following expressions:

(i) $\left(3+\sqrt{3}\right)\left(2+\sqrt{2}\right)$ (ii) $\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)$

(iii) $\left(\sqrt{5}+\sqrt{2}\right)^2$ (iv) $\left(\sqrt{5}-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{2}\right)$

Answer:

(i) $\left(3+\sqrt{3}\right)\left(2+\sqrt{2}\right)=3\left(2+\sqrt{2}\right)+\sqrt{3}\left(2+\sqrt{2}\right)$

$=6+3\sqrt{2}+2\sqrt{3}+\sqrt{6}$

(ii) $\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right) \;=(3)^2-\left(\sqrt{3}\right)^2$

= 9 − 3 = 6

(iii) $\left(\sqrt{5}+\sqrt{2}\right)^2=\left(\sqrt{5}\right)^2+\left(\sqrt{2}\right)^2+2\left(\sqrt{5}\right)\left(\sqrt{2}\right)$

$=5+2+2\sqrt{10}=7+2\sqrt{10}$

(iv) $\left(\sqrt{5}-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{2}\right)=\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2$

= 5 − 2 = 3

Question 3:

Recall, π is defined as the ratio of the circumference (say *c*) of a circle to its diameter (say *d*). That is, $\pi = \frac{c}{d}$. This seems to contradict the fact that π is irrational. How will you resolve this contradiction?

Answer:

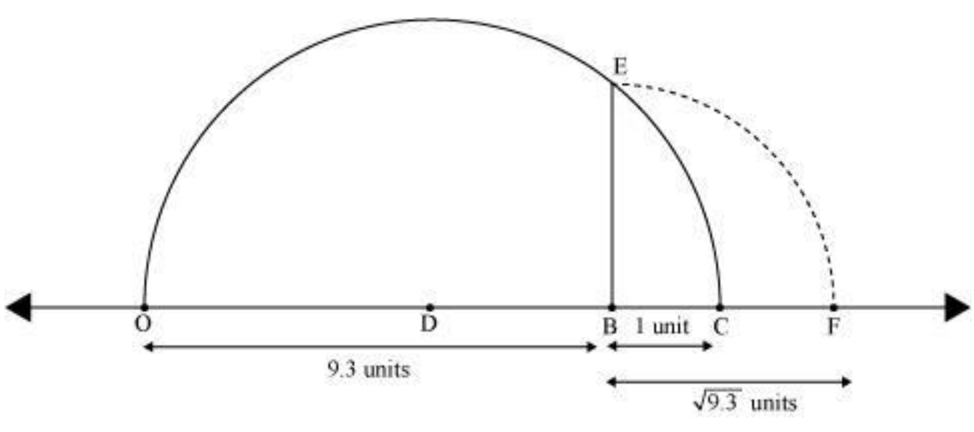
There is no contradiction. When we measure a length with scale or any other instrument, we only obtain an approximate rational value. We never obtain an exact value. For this reason, we may not realize that either *c* or *d* is irrational. Therefore, the fraction $\frac{c}{d}$ is irrational. Hence, π is irrational.

Question 4:

Represent $\sqrt{9.3}$ on the number line.

Answer:

Mark a line segment OB = 9.3 on number line. Further, take BC of 1 unit. Find the mid-point D of OC and draw a semi-circle on OC while taking D as its centre. Draw a perpendicular to line OC passing through point B. Let it intersect the semi-circle at E. Taking B as centre and BE as radius, draw an arc intersecting number line at F. BF is $\sqrt{9.3}$.



Question 5:

Rationalise the denominators of the following:

(i) $\frac{1}{\sqrt{7}}$ (ii) $\frac{1}{\sqrt{7}-\sqrt{6}}$

(iii) $\frac{1}{\sqrt{5}+\sqrt{2}}$ (iv) $\frac{1}{\sqrt{7}-2}$

Answer:

(i) $\frac{1}{\sqrt{7}}=\frac{1\times\sqrt{7}}{1\times\sqrt{7}}=\frac{\sqrt{7}}{7}$

(ii) $\frac{1}{\sqrt{7}-\sqrt{6}}=\frac{1\quad\left(\sqrt{7}+\sqrt{6}\right)}{\left(\sqrt{7}-\sqrt{6}\right)\left(\sqrt{7}+\sqrt{6}\right)}$

$$=\frac{\sqrt{7}+\sqrt{6}}{\left(\sqrt{7}\right)^2-\left(\sqrt{6}\right)^2}$$

$$=\frac{\sqrt{7}+\sqrt{6}}{7-6}=\frac{\sqrt{7}+\sqrt{6}}{1}=\sqrt{7}+\sqrt{6}$$

(iii) $\frac{1}{\sqrt{5}+\sqrt{2}}=\frac{1\quad\left(\sqrt{5}-\sqrt{2}\right)}{\left(\sqrt{5}+\sqrt{2}\right)\left(\sqrt{5}-\sqrt{2}\right)}$

$$=\frac{\sqrt{5}-\sqrt{2}}{\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2}=\frac{\sqrt{5}-\sqrt{2}}{5-2}$$

$$=\frac{\sqrt{5}-\sqrt{2}}{3}$$

(iv) $\frac{1}{\sqrt{7}-2}=\frac{1\quad\left(\sqrt{7}+2\right)}{\left(\sqrt{7}-2\right)\left(\sqrt{7}+2\right)}$

$$= \frac{\sqrt{7}+2}{\left(\sqrt{7}\right)^2-(2)^2}$$

$$= \frac{\sqrt{7}+2}{7-4} = \frac{\sqrt{7}+2}{3}$$